Homework: Chain Rule

Calculate $\frac{dy}{dx}$.

228. $y = (3x^2+3x-1)^4$

Sol: $\begin{cases} f(u) = u^4, & f'(u) = 4u^3 \\ u(x) = 3x^2+3x-1 & u'(x) = 6x+3 \end{cases}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}[f(u(x))] = f'(u(x)) \cdot u'(x) = 4(3x^2+3x-1)^3(6x+3)$ ▨

232. $y = \frac{1}{\sin^2(x)}$

Sol: $\begin{cases} f(u) = \frac{1}{u^2} = u^{-2}, & f'(u) = -2u^{-3} \\ u(x) = \sin(x), & u'(x) = \cos(x) \end{cases}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}[f(u(x))] = f'(u(x)) \cdot u'(x) = -2(\sin(x))^{-3} \cdot \cos(x) = \frac{-2\cos(x)}{\sin^3(x)}$ ▨

234. $y = x^2\cos^4(x)$

Sol: $\frac{dy}{dx} = \frac{d}{dx}[x^2\cos^4(x)] = \frac{d}{dx}[x^2]\cos^4(x) + x^2\underbrace{\frac{d}{dx}[\cos^4(x)]}$

Chain Rule!

$\begin{cases} f(u) = u^4 & f'(u) = 4u^3 \\ u(x) = \cos(x) & u'(x) = -\sin(x) \end{cases}$

$= 2x\cos^4(x) + x^2 \cdot 4\cos^3(x)(-\sin(x))$

$= 2x\cos^4(x) - 4x^2\cos^3(x)\sin(x)$

$= 2x\cos^3(x)(\cos(x) - 2x\sin(x))$ ▨