

HOMEWORK: TRIG DERIVATIVES

182. Calculate $\frac{d}{dx} \left[\frac{\tan(x)}{1-\sec(x)} \right]$

Sol A: Derive as given.

$$\begin{aligned} \frac{d}{dx} \left[\frac{\tan(x)}{1-\sec(x)} \right] &= \frac{(1-\sec(x)) \frac{d}{dx} [\tan(x)] - \tan(x) \frac{d}{dx} [1-\sec(x)]}{(1-\sec(x))^2} \\ &= \frac{(1-\sec(x)) \sec^2(x) - \tan(x) (\sec(x) \tan(x))}{(1-\sec(x))^2} \\ &= \frac{\sec(x) (\sec(x) - \sec^2(x) - \tan^2(x))}{(1-\sec(x))^2} \quad \square \end{aligned}$$

Sol B: First simplify: $\frac{\tan(x)}{1-\sec(x)} = \frac{\frac{\sin(x)}{\cos(x)}}{1-\frac{1}{\cos(x)}} \cdot \frac{\cos(x)}{\cos(x)} = \frac{\sin(x)}{\cos(x)-1}$ (if $\cos(x) \neq 0$)

$$\begin{aligned} \frac{d}{dx} \left[\frac{\tan(x)}{1-\sec(x)} \right] &= \frac{d}{dx} \left[\frac{\sin(x)}{\cos(x)-1} \right] \\ &= \frac{(\cos(x)-1) \frac{d}{dx} [\sin(x)] - \sin(x) \frac{d}{dx} [\cos(x)-1]}{(\cos(x)-1)^2} \\ &= \frac{(\cos(x)-1) \cos(x) - \sin(x) (-\sin(x))}{(\cos(x)-1)^2} \\ &= \frac{\cos^2(x) - \cos(x) + \sin^2(x)}{(\cos(x)-1)^2} \\ &= \frac{1 - \cos(x)}{(\cos(x)-1)^2} = \frac{-(\cos(x)-1)}{(\cos(x)-1)^2} = \frac{-1}{\cos(x)-1} \quad \square \end{aligned}$$

186. Calculate an equation of the tangent line to $f(x) = \csc(x)$ at $x = \frac{\pi}{2}$.

Sol: $y - y_0 = m(x - x_0)$

$$f'(x) = \frac{d}{dx} [\csc(x)] = -\csc(x) \cot(x), \quad m = f'\left(\frac{\pi}{2}\right) = -\csc\left(\frac{\pi}{2}\right) \cot\left(\frac{\pi}{2}\right) = -1 \cdot 0 = 0$$

$$x_0 = \frac{\pi}{2}, \quad y_0 = f\left(\frac{\pi}{2}\right) = \csc\left(\frac{\pi}{2}\right) = 1 \quad \therefore y - 1 = 0(x - \frac{\pi}{2}) \quad \text{or} \quad y = 1 \quad \square$$

192. Calculate $\frac{d^2}{dx^2} [\sin(x) \cos(x)]$.

Sol: $y = \sin(x) \cos(x)$

$$\frac{dy}{dx} = \frac{d}{dx} [\sin(x) \cos(x)] = \frac{d}{dx} [\sin(x)] \cos(x) + \sin(x) \frac{d}{dx} [\cos(x)] = \cos^2(x) - \sin^2(x)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx} [\cos^2(x) - \sin^2(x)] = 2\cos(x) \cdot (-\sin(x)) - 2\sin(x) \cdot \cos(x) = -4\sin(x) \cos(x) \quad \square$$