# Homework: Trig Derivatives

182. Calculate $\frac{d}{dx}\left[\frac{\tan(x)}{1-\sec(x)}\right]$

Sol A: Derive as given.

$$\frac{d}{dx}\left[\frac{\tan(x)}{1-\sec(x)}\right] = \frac{(1-\sec(x))\frac{d}{dx}[\tan(x)] - \tan(x)\frac{d}{dx}[1-\sec(x)]}{(1-\sec(x))^2}$$

$$= \frac{(1-\sec(x))\sec^2(x) - \tan(x)(\sec(x)\tan(x))}{(1-\sec(x))^2}$$

$$= \frac{\sec(x)(\sec(x)-\sec^2(x)-\tan^2(x))}{(1-\sec(x))^2} \quad \blacksquare$$

Sol B: First simplify: $\frac{\tan(x)}{1-\sec(x)} = \frac{\frac{\sin(x)}{\cos(x)}}{1-\frac{1}{\cos(x)}}\cdot\frac{\cos(x)}{\cos(x)} = \frac{\sin(x)}{\cos(x)-1}$ (if $\cos(x)\neq 0$)

$$\frac{d}{dx}\left[\frac{\tan(x)}{1-\sec(x)}\right] = \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)-1}\right]$$

$$= \frac{(\cos(x)-1)\frac{d}{dx}[\sin(x)] - \sin(x)\frac{d}{dx}[\cos(x)-1]}{(\cos(x)-1)^2}$$

$$= \frac{(\cos(x)-1)\cos(x) - \sin(x)(-\sin(x))}{(\cos(x)-1)^2}$$

$$= \frac{\cos^2(x)-\cos(x)+\sin^2(x)}{(\cos(x)-1)^2}$$

$$= \frac{1-\cos(x)}{(\cos(x)-1)^2} = \frac{-(\cos(x)-1)}{(\cos(x)-1)^2} = \frac{-1}{\cos(x)-1} \quad \blacksquare$$

186. Calculate an equation of the tangent line to $f(x)=\csc(x)$ at $x=\frac{\pi}{2}$.

Sol: $y-y_0 = m(x-x_0)$

$f'(x) = \frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$, $m = f'(\frac{\pi}{2}) = -\csc(\frac{\pi}{2})\cot(\frac{\pi}{2}) = -1\cdot 0 = 0$

$x_0 = \frac{\pi}{2}$, $y_0 = f(\frac{\pi}{2}) = \csc(\frac{\pi}{2}) = 1$ $\therefore y-1 = 0(x-\frac{\pi}{2})$ or $y=1$ $\blacksquare$

192. Calculate $\frac{d^2}{dx^2}[\sin(x)\cos(x)]$.

Sol: $y = \sin(x)\cos(x)$

$$\frac{dy}{dx} = \frac{d}{dx}[\sin(x)\cos(x)] = \frac{d}{dx}[\sin(x)]\cos(x) + \sin(x)\cdot\frac{d}{dx}[\cos(x)] = \cos^2(x)-\sin^2(x)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}[\cos^2(x)-\sin^2(x)] = 2\cos(x)\cdot -\sin(x) - 2\sin(x)\cdot\cos(x) = -4\sin(x)\cos(x) \quad \blacksquare$$