# Derivative Rules Homework

106-116 evens: Calculate $f'(x)$.

106. $f(x) = x^7 + 10$

Sol: $f'(x) = \frac{d}{dx}[x^7+10] = \frac{d}{dx}[x^7] + \frac{d}{dx}[10] = 7x^6 + 0 = 7x^6$ ▨

108. $f(x) = 4x^2 - 7x$

Sol: $f'(x) = \frac{d}{dx}[4x^2-7x] = \frac{d}{dx}[4x^2] - \frac{d}{dx}[7x]$
$= 4\frac{d}{dx}[x^2] - 7\frac{d}{dx}[x] = 4 \cdot 2x - 7 \cdot 1 = 8x - 7$ ▨

110. $f(x) = x^4 + \frac{2}{x} = x^4 + 2x^{-1}$

Sol: $f'(x) = \frac{d}{dx}[x^4+2x^{-1}] = \frac{d}{dx}[x^4] + \frac{d}{dx}[2x^{-1}]$
$= \frac{d}{dx}[x^4] + 2\frac{d}{dx}[x^{-1}] = 4x^3 + 2\cdot(-1)x^{-2} = 4x^3 - 2x^{-2}$ ▨

112. $f(x) = (x+2)(2x^2-3)$

Sol A: $f'(x) = \frac{d}{dx}[(x+2)(2x^2-3)] = \frac{d}{dx}[x+2](2x^2-3) + (x+2)\frac{d}{dx}[2x^2-3]$
$= \left(\frac{d}{dx}[x] + \frac{d}{dx}[2]\right)(2x^2-3) + (x+2)\left(\frac{d}{dx}[2x^2] - \frac{d}{dx}[3]\right)$
$= \left(\frac{d}{dx}[x] + \frac{d}{dx}[2]\right)(2x^2-3) + (x+2)\left(2\frac{d}{dx}[x^2] - \frac{d}{dx}[3]\right)$
$= (1+0)(2x^2-3) + (x+2)(2(2x) - 0)$
$= 2x^2 - 3 + (x+2)\cdot 4x$
$= 2x^2 - 3 + 4x^2 + 8x = 6x^2 + 8x - 3$ ▨

Sol B: $f(x) = (x+2)(2x^2-3) = 2x^3 - 3x + 4x^2 - 6 = 2x^3 + 4x^2 - 3x - 6$

$f'(x) = \frac{d}{dx}[2x^3+4x^2-3x-6] = \frac{d}{dx}[2x^3] + \frac{d}{dx}[4x^2] - \frac{d}{dx}[3x] - \frac{d}{dx}[6]$
$= 2\frac{d}{dx}[x^3] + 4\frac{d}{dx}[x^2] - 3\frac{d}{dx}[x] - \frac{d}{dx}[6] = 2\cdot 3x^2 + 4\cdot 2x - 3\cdot 1 - 0$
$= 6x^2 + 8x - 3$ ▨

114. $f(x) = \dfrac{x^3+2x^2-4}{3} = \frac{1}{3}x^3 + \frac{2}{3}x^2 - \frac{4}{3}$

<u>Sol</u>: $f'(x) = \frac{d}{dx}\left[\frac{1}{3}x^3 + \frac{2}{3}x^2 - \frac{4}{3}\right] = \frac{d}{dx}\left[\frac{1}{3}x^3\right] + \frac{d}{dx}\left[\frac{2}{3}x^2\right] - \frac{d}{dx}\left[\frac{4}{3}\right]$

$$= \frac{1}{3}\frac{d}{dx}[x^3] + \frac{2}{3}\frac{d}{dx}[x^2] - \frac{d}{dx}\left[\frac{4}{3}\right] = \frac{1}{3}\cdot 3x^2 + \frac{2}{3}\cdot 2x - 0$$

$$= x^2 + \frac{4}{3}x \quad \blacksquare$$

116. $f(x) = \dfrac{x^2+4}{x^2-4}$

<u>Sol</u>: $f'(x) = \frac{d}{dx}\left[\frac{x^2+4}{x^2-4}\right] = \dfrac{(x^2-4)\frac{d}{dx}[x^2+4] - (x^2+4)\frac{d}{dx}[x^2-4]}{(x^2-4)^2}$

$$= \frac{(x^2-4)(2x) - (x^2+4)(2x)}{(x^2-4)^2} = \frac{-16x}{(x^2-4)^2} \quad \blacksquare$$

128. The functions $f$ and $g$ are differentiable and have the values given in the table. Calculate $h'(3)$ if $h(x) = 2x + f(x)g(x)$.

| $x$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $f(x)$ | 3 | 5 | −2 | 0 |
| $g(x)$ | 2 | 3 | −4 | 6 |
| $f'(x)$ | −1 | 7 | 8 | −3 |
| $g'(x)$ | 4 | 1 | 2 | 9 |

<u>Sol</u>: $h'(x) = \frac{d}{dx}[2x + f(x)g(x)] = \frac{d}{dx}[2x] + \frac{d}{dx}[f(x)g(x)]$

$$= 2\frac{d}{dx}[x] + \left(f'(x)g(x) + f(x)g'(x)\right)$$

$$= 2\cdot 1 + f'(x)g(x) + f(x)g'(x)$$

$$\therefore h'(3) = 2 + f'(3)g(3) + f(3)g'(3) = 2 + 8\cdot(-4) + (-2)\cdot 2 = 2 - 32 - 4 = -34 \quad \blacksquare$$

<u>PROB</u>: Calculate $\frac{d}{dx}[\tan(x)]$.

<u>Sol</u>: $\frac{d}{dx}[\tan(x)] = \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right]$

$$= \frac{\cos(x)\frac{d}{dx}[\sin(x)] - \sin(x)\frac{d}{dx}[\cos(x)]}{(\cos(x))^2}$$

$$= \frac{\cos(x)\cdot\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x) \quad \blacksquare$$