Calculate derivatives of each of the trigonometric ratios, given that you know $\frac{d}{dx}[\sin(x)] = \cos(x)$ and $\frac{d}{dx}[\cos(x)] = -\sin(x)$.

Sol: We'll calculate derivatives of Ⓐ $\tan(x)$, Ⓑ $\sec(x)$, Ⓒ $\csc(x)$, and Ⓓ $\cot(x)$.

Ⓐ
$$\frac{d}{dx}[\tan(x)] = \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right] = \frac{\cos(x)\frac{d}{dx}[\sin(x)] - \sin(x)\frac{d}{dx}[\cos(x)]}{(\cos(x))^2}$$
$$= \frac{\cos(x)\cdot\cos(x) - \sin(x)\cdot(-\sin(x))}{\cos^2(x)}$$
$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \sec^2(x)$$

Ⓑ
$$\frac{d}{dx}[\sec(x)] = \frac{d}{dx}\left[\frac{1}{\cos(x)}\right] = \frac{\cos(x)\frac{d}{dx}[1] - 1\cdot\frac{d}{dx}[\cos(x)]}{(\cos(x))^2}$$
$$= \frac{\cos(x)\cdot 0 - (-\sin(x))}{\cos^2(x)}$$
$$= \frac{\sin(x)}{\cos^2(x)} = \frac{1}{\cos(x)}\cdot\frac{\sin(x)}{\cos(x)} = \sec(x)\tan(x)$$

Ⓒ
$$\frac{d}{dx}[\csc(x)] = \frac{d}{dx}\left[\frac{1}{\sin(x)}\right] = \frac{\sin(x)\cdot\frac{d}{dx}[1] - 1\cdot\frac{d}{dx}[\sin(x)]}{(\sin(x))^2}$$
$$= \frac{\sin(x)\cdot 0 - \cos(x)}{\sin^2(x)}$$
$$= -\frac{\cos(x)}{\sin^2(x)} = -\frac{1}{\sin(x)}\cdot\frac{\cos(x)}{\sin(x)} = -\csc(x)\cot(x)$$

Ⓓ
$$\frac{d}{dx}[\cot(x)] = \frac{d}{dx}\left[\frac{\cos(x)}{\sin(x)}\right] = \frac{\sin(x)\frac{d}{dx}[\cos(x)] - \cos(x)\frac{d}{dx}[\sin(x)]}{(\sin(x))^2}$$
$$= \frac{\sin(x)\cdot(-\sin(x)) - \cos(x)\cdot\cos(x)}{\sin^2(x)}$$
$$= -\left(\frac{\sin^2(x) + \cos^2(x)}{\sin^2(x)}\right)$$
$$= -\frac{1}{\sin^2(x)} = -\csc^2(x)$$

Derivatives of Trigonometric Functions

175–184: Calculate $\frac{dy}{dx}$.

175. $y = x^2 - \sec(x) + 1$

Sol: $\frac{dy}{dx} = \frac{d}{dx}\left[x^2 - \sec(x) + 1\right] = 2x - \sec(x)\tan(x)$ $\boxed{\checkmark}$

176. $y = 3\csc(x) + \frac{5}{x}$

Sol: $\frac{dy}{dx} = \frac{d}{dx}\left[3\csc(x) + 5x^{-1}\right] = -3\csc(x)\cot(x) - 5x^{-2}$ $\boxed{\checkmark}$

177. $y = x^2\cot(x)$

Sol: $\frac{dy}{dx} = \frac{d}{dx}\left[x^2\cot(x)\right] = \frac{d}{dx}\left[x^2\right]\cot(x) + x^2\frac{d}{dx}\left[\cot(x)\right]$

$= 2x\cot(x) - x^2\csc^2(x)$ $\boxed{\checkmark}$

178. $y = x - x^3\sin(x)$

Sol: $\frac{dy}{dx} = \frac{d}{dx}\left[x - x^3\sin(x)\right] = 1 - \left(\frac{d}{dx}\left[x^3\right]\sin(x) + x^3\frac{d}{dx}\left[\sin(x)\right]\right)$

$= 1 - 3x^2\sin(x) - x^3\cos(x)$ $\boxed{\checkmark}$

179. $y = \frac{\sec(x)}{x}$

Sol: $\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\sec(x)}{x}\right] = \frac{x\frac{d}{dx}\left[\sec(x)\right] - \sec(x)\frac{d}{dx}\left[x\right]}{x^2}$

$= \frac{x\sec(x)\tan(x) - \sec(x)}{x^2}$ $\boxed{\checkmark}$

180. $y = \sin(x)\tan(x)$

Sol: $\frac{dy}{dx} = \frac{d}{dx}[\sin(x)\tan(x)] = \frac{d}{dx}[\sin(x)]\tan(x) + \sin(x)\frac{d}{dx}[\tan(x)]$

$= \cos(x)\tan(x) + \sin(x)\cdot\sec^2(x)$

$= \cos(x)\cdot\frac{\sin(x)}{\cos(x)} + \sin(x)\sec^2(x)$

$= \sin(x)(1+\sec^2(x))$ $\boxed{/\!/}$

181. $y = (x+\cos(x))(1-\sin(x))$

Sol: $\frac{dy}{dx} = \frac{d}{dx}[(x+\cos(x))(1-\sin(x))]$

$= \frac{d}{dx}[x+\cos(x)](1-\sin(x)) + (x+\cos(x))\frac{d}{dx}[1-\sin(x)]$

$= (1-\sin(x))(1-\sin(x)) + (x+\cos(x))\cdot -\cos(x)$

$= 1-2\sin(x)+\sin^2(x) - x\cos(x) - \cos^2(x)$ $\boxed{/\!/}$

182. $y = \frac{\tan(x)}{1-\sec(x)}$

Sol 1: $\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\tan(x)}{1-\sec(x)}\right] = \frac{(1-\sec(x))\frac{d}{dx}[\tan(x)] - \tan(x)\frac{d}{dx}[1-\sec(x)]}{(1-\sec(x))^2}$

$= \frac{(1-\sec(x))\sec^2(x) - \tan(x)\cdot -\sec(x)\tan(x)}{(1-\sec(x))^2}$

$= \frac{\sec(x)(\sec(x)-\sec^2(x)+\tan^2(x))}{(1-\sec(x))^2}$ $\boxed{/\!/}$

Sol 2: $\frac{dy}{dx} = \frac{d}{dx}\left[\frac{\tan(x)}{1-\sec(x)}\right] = \frac{d}{dx}\left[\frac{\sin(x)/\cos(x)}{1-1/\cos(x)}\cdot\frac{\cos(x)}{\cos(x)}\right] = \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)-1}\right]$

$= \frac{(\cos(x)-1)\frac{d}{dx}[\sin(x)] - \sin(x)\frac{d}{dx}[\cos(x)-1]}{(\cos(x)-1)^2}$

$= \frac{(\cos(x)-1)\cos(x) - \sin(x)\cdot -\sin(x)}{(\cos(x)-1)^2} = \frac{\cos^2(x)-\cos(x)+\sin^2(x)}{(\cos(x)-1)^2}$

$= \frac{1-\cos(x)}{(\cos(x)-1)^2} = \frac{1}{1-\cos(x)}$ $\boxed{/\!/}$

183. $y = \dfrac{1-\cot(x)}{1+\cot(x)}$

Sol 1: $$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{1-\cot(x)}{1+\cot(x)}\right] = \frac{(1+\cot(x))\frac{d}{dx}[1-\cot(x)] - (1-\cot(x))\frac{d}{dx}[1+\cot(x)]}{(1+\cot(x))^2}$$

$$= \frac{(1+\cot(x))\cdot -(-\csc^2(x)) - (1-\cot(x))\cdot -\csc^2(x)}{(1+\cot(x))^2}$$

$$= \frac{\csc^2(x)\left(1+\cot(x) - 1 + \cot(x)\right)}{(1+\cot(x))^2} = \frac{2\csc^2(x)\cot(x)}{(1+\cot(x))^2} \boxed{\text{S}}$$

Sol 2: $$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{1-\cot(x)}{1+\cot(x)}\right] = \frac{d}{dx}\left[\frac{1-\frac{\cos(x)}{\sin(x)}}{1+\frac{\cos(x)}{\sin(x)}}\cdot\frac{\sin(x)}{\sin(x)}\right] = \frac{d}{dx}\left[\frac{\sin(x)-\cos(x)}{\sin(x)+\cos(x)}\right]$$

$$= \frac{(\sin(x)+\cos(x))\frac{d}{dx}[\sin(x)-\cos(x)] - (\sin(x)-\cos(x))\frac{d}{dx}[\sin(x)+\cos(x)]}{(\sin(x)+\cos(x))^2}$$

$$= \frac{(\sin(x)+\cos(x))(\cos(x)+\sin(x)) - (\sin(x)-\cos(x))(\cos(x)-\sin(x))}{(\sin(x)+\cos(x))^2}$$

$$= \frac{(\sin(x)+\cos(x))^2 + (\sin(x)-\cos(x))^2}{(\sin(x)+\cos(x))^2}$$

$$= \frac{\sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x) + \sin^2(x) - 2\sin(x)\cos(x) + \cos^2(x)}{(\sin(x)+\cos(x))^2}$$

$$= \frac{2(\sin^2(x)+\cos^2(x))}{(\sin(x)+\cos(x))^2} = \frac{2}{(\sin(x)+\cos(x))^2} \quad \boxed{/\!/}$$

184. $y = \cos(x)\,(1+\csc(x))$

Sol: $$\frac{dy}{dx} = \frac{d}{dx}\left[\cos(x)(1+\csc(x))\right] = \frac{d}{dx}[\cos(x)](1+\csc(x)) + \cos(x)\frac{d}{dx}[1+\csc(x)]$$

$$= -\sin(x)\cdot(1+\csc(x)) + \cos(x)\left(-\csc(x)\cot(x)\right) \leftarrow \text{ok}$$

$$= -\sin(x)\left(1+\frac{1}{\sin(x)}\right) - \cos(x)\left(\frac{1}{\sin(x)}\cdot\frac{\cos(x)}{\sin(x)}\right)$$

$$= -\sin(x) - (1+\cot^2(x)) \quad \leftarrow 1+\cot^2(x) = \csc^2(x)$$

$$= -\sin(x) - \csc^2(x) \quad \boxed{/\!/}$$

185–190: Find the equation of the tangent line to $y=f(x)$ at the given value of x.

Recall: ① $f'(a)$ is the slope of the tangent line at a.
② Lines have point-slope form $y-y_0=m(x-x_0)$.
③ I give my answers in both point-slope form and slope intercept form, but you can choose to only give one (usually point-slope is easier).

185. $f(x)=-\sin(x)$ at $x=0$

Sol: $f'(x)=\frac{d}{dx}\left[-\sin(x)\right]=-\frac{d}{dx}\left[\sin(x)\right]=-\cos(x)$.

$\therefore\ m=f'(0)=-\cos(0)=-1$

$f(0)=-\sin(0)=-0=0$

$\therefore$ tangent line is $y-0=-1(x-0)$ OR $y=-x$.

186. $f(x)=\csc(x)$ at $x=\frac{\pi}{2}$

Sol: $f'(x)=\frac{d}{dx}\left[\csc(x)\right]=-\csc(x)\cot(x)$

$\therefore\ m=f'\left(\frac{\pi}{2}\right)=-\csc\left(\frac{\pi}{2}\right)\cot\left(\frac{\pi}{2}\right)=-\frac{1}{\sin\left(\frac{\pi}{2}\right)}\cdot\frac{\cos\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)}=-\frac{1}{1}\cdot\frac{0}{1}=0$

$f\left(\frac{\pi}{2}\right)=\csc\left(\frac{\pi}{2}\right)=\frac{1}{\sin(\pi/2)}=\frac{1}{1}=1$

$\therefore\ y-1=0\left(x-\frac{\pi}{2}\right)$ OR $y=1$

187. $f(x)=1+\cos(x)$ at $x=\frac{3\pi}{2}$

Sol: $f'(x)=\frac{d}{dx}\left[1+\cos(x)\right]=-\sin(x)$

$m=f'\left(\frac{3\pi}{2}\right)=-\sin\left(\frac{3\pi}{2}\right)=-(-1)=1$

$f\left(\frac{3\pi}{2}\right)=1+\cos\left(\frac{3\pi}{2}\right)=1+0=1$

$\therefore\ y-1=1\left(x-\frac{3\pi}{2}\right)$ OR $y=x+1-\frac{3\pi}{2}$

188. $f(x) = \sec(x)$ at $x = \frac{\pi}{4}$

Sol: $f'(x) = \sec(x)\tan(x)$

$$m = f'\left(\frac{\pi}{4}\right) = \sec\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{4}\right) = \frac{1}{\cos(\pi/4)} \cdot \frac{\sin(\pi/4)}{\cos(\pi/4)} = \frac{\cancel{\sqrt{2}/2}}{(\sqrt{2}/2)(\cancel{\sqrt{2}/2})} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$f\left(\frac{\pi}{4}\right) = \sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos(\pi/4)} = \frac{1}{\sqrt{2}/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\therefore\ y - \sqrt{2} = \sqrt{2}\left(x - \frac{\pi}{4}\right) \quad \text{OR} \quad y = \sqrt{2}x + \sqrt{2}\left(1 - \frac{\pi}{4}\right)$$

189. $f(x) = x^2 - \tan(x)$ at $x = 0$

Sol: $f'(x) = 2x - \sec^2(x)$

$$m = f'(0) = 2 \cdot 0 - \sec^2(0) = -\frac{1}{\cos^2(0)} = -\frac{1}{1^2} = -1$$

$$f(0) = 0^2 - \tan(0) = 0 - 0 = 0$$

$$\therefore\ y - 0 = -1(x - 0) \quad \text{OR} \quad y = -x$$

190. $f(x) = 5\cot(x)$ at $x = \frac{\pi}{4}$

Sol: $f'(x) = -5\csc^2(x)$

$$m = -5\csc^2\left(\frac{\pi}{4}\right) = -5 \cdot \frac{1}{\sin^2\left(\frac{\pi}{4}\right)} = -5/\left(\sqrt{2}/2\right)^2 = -5/(2/4) = -10$$

$$f\left(\frac{\pi}{4}\right) = 5\cot\left(\frac{\pi}{4}\right) = 5 \cdot \frac{\cos(\pi/4)}{\sin(\pi/4)} = 5 \cdot \frac{\sqrt{2}/2}{\sqrt{2}/2} = 5 \cdot 1 = 5$$

$$\therefore\ y - 5 = -10\left(x - \frac{\pi}{4}\right) \quad \text{OR} \quad y = -10x + 5\left(\frac{\pi}{2} + 1\right)$$

191–196: Calculate $\frac{d^2y}{dx^2}$ for the given $y = f(x)$.

Recall: $\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{d}{dx}[y]\right]$ is the second derivative.

191. $y = x\sin(x) - \cos(x)$

Sol: $\frac{dy}{dx} = \frac{d}{dx}\left[x\sin(x) - \cos(x)\right]$

$= x\cos(x) + 1\cdot\sin(x) - (-\sin(x))$

$= x\cos(x) + 2\sin(x)$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[x\cos(x) + 2\sin(x)\right]$

$= x(-\sin(x)) + 1\cdot\cos(x) + 2\cos(x)$

$= 3\cos(x) - x\sin(x)$ $\boxed{/\!/}$

192. $y = \sin(x)\cos(x)$

Sol: $\frac{dy}{dx} = \frac{d}{dx}\left[\sin(x)\cos(x)\right]$

$= \cos(x)\cdot\cos(x) + \sin(x)\cdot(-\sin(x))$

$= \cos^2(x) - \sin^2(x)$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\cos^2(x) - \sin^2(x)\right]$ generalized power rule, twice

$= 2\cos(x)\cdot(-\sin(x)) - 2\sin(x)\cdot\cos(x)$

$= -4\sin(x)\cos(x)$ $\boxed{/\!/}$

193. $y = x - \frac{1}{2}\sin(x)$

Sol: $\frac{dy}{dx} = \frac{d}{dx}\left[x - \frac{1}{2}\sin(x)\right] = 1 - \frac{1}{2}\cdot\cos(x)$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[1 - \frac{1}{2}\cos(x)\right] = 0 - \frac{1}{2}(-\sin(x)) = \frac{1}{2}\sin(x)$ $\boxed{/\!/}$

194. $y = \frac{1}{x} + \tan(x) = x^{-1} + \tan(x)$

Sol: $\frac{dy}{dx} = \frac{d}{dx}\left[x^{-1} + \tan(x)\right] = -x^{-2} + \sec^2(x)$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[-x^{-2} + \sec^2(x)\right] = -(-2x^{-3}) + 2\sec(x)\cdot\sec(x)\tan(x)$

$= 2x^{-3} + 2\sec^2(x)\tan(x)$ ☑

195. $y = 2\csc(x)$

Sol: $\frac{dy}{dx} = \frac{d}{dx}\left[2\csc(x)\right] = 2\cdot -\csc(x)\cot(x) = -2\csc(x)\cot(x).$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[-2\csc(x)\cot(x)\right] = -2\left(\csc(x)\frac{d}{dx}\left[\cot(x)\right] + \frac{d}{dx}\left[\csc(x)\right]\cot(x)\right)$

$= -2\left(\csc(x)\cdot -\csc^2(x) + (-\csc(x)\cot(x))\cot(x)\right)$

$= 2\csc(x)\left(\csc^2(x) + \cot^2(x)\right)$ ☑

196. $y = \sec^2(x)$

Sol: $\frac{dy}{dx} = \frac{d}{dx}\left[\sec^2(x)\right] = 2\sec(x)\cdot\sec(x)\tan(x) = 2\sec^2(x)\tan(x)$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[2\sec^2(x)\tan(x)\right] = 2\left(\sec^2(x)\frac{d}{dx}\left[\tan(x)\right] + \frac{d}{dx}\left[\sec^2(x)\right]\tan(x)\right)$

$= 2\left(\sec^2(x)\cdot\sec^2(x) + 2\sec^2(x)\tan(x)\cdot\tan(x)\right)$

$= 2\sec^2(x)\left(\sec^2(x) + 2\tan^2(x)\right)$ ☑

200. A mass on a spring bounces up and down in simple harmonic motion, modeled by the function $S(t) = -6\cos(t)$ where S is measured in inches and t is measured in seconds. Calculate the rate at which the spring is oscillating after 5 seconds.

Sol: The desired quantity is $S'(5)$

$$S'(t) = \frac{d}{dt}[-6\cos(t)] = -6 \cdot -\sin(t) = 6\sin(t).$$

$$S'(5) = 6\sin(5) \leftarrow \text{exact value!}$$

$\therefore$ the spring oscillates at a rate of $6\sin(5)$ inches/sec. $\boxtimes$

201. Let the position of a swinging pendulum in simple harmonic motion be given by $S(t) = a\cos(t) + b\sin(t)$ where a and b are constants, t measures time in seconds, and S measures position in centimeters. If the position is 0 cm and the velocity is 3 cm/s when $t=0$, calculate a and b.

Sol: We are told: $\begin{cases} S(0) = 0 \\ S'(0) = 3 \end{cases}$

$$\therefore \quad 0 = S(0) = a\cos(0) + b\sin(0) = a \cdot 1 + b \cdot 0 = a$$

$$S'(t) = \frac{d}{dt}[a\cos(t) + b\sin(t)]$$
$$= a(-\sin(t)) + b\cos(t)$$
$$= b\cos(t) - a\sin(t)$$

$$3 = S'(0) = b\cos(0) - a\sin(0) = b \cdot 1 - a \cdot 0 = b$$

$\therefore \quad a = 0$ and $b = 3$ $\boxtimes$

209–213. Calculate the given higher-order derivative.

209. $\frac{d^3y}{dx^3}$ for $y = 3\cos(x)$.

Sol: $\frac{dy}{dx} = -3\sin(x)$, $\quad \frac{d^2y}{dx^2} = \frac{d}{dx}\left[-3\sin(x)\right] = -3\cos(x)$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left[-3\cos(x)\right] = 3\sin(x) \quad \blacksquare$$

210. $\frac{d^2y}{dx^2}$ for $y = 3\sin(x) + x^2\cos(x)$

Sol:
$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left[3\sin(x) + x^2\cos(x)\right] \\
&= \frac{d}{dx}\left[3\sin(x)\right] + \frac{d}{dx}\left[x^2\cos(x)\right] \\
&= 3\cos(x) + \frac{d}{dx}\left[x^2\right]\cos(x) + x^2\frac{d}{dx}\left[\cos(x)\right] \\
&= 3\cos(x) + 2x\cos(x) + x^2(-\sin(x)) \\
&= (2x+3)\cos(x) - x^2\sin(x)
\end{aligned}$$

$$\begin{aligned}
\frac{d^2y}{dx^2} &= \frac{d}{dx}\left[(2x+3)\cos(x) - x^2\sin(x)\right] \\
&= \frac{d}{dx}\left[(2x+3)\cos(x)\right] - \frac{d}{dx}\left[x^2\sin(x)\right] \\
&= \left(\frac{d}{dx}\left[2x+3\right]\cos(x) + \frac{d}{dx}\left[\cos(x)\right](2x+3)\right) \\
&\quad - \left(\frac{d}{dx}\left[x^2\right]\sin(x) + \frac{d}{dx}\left[\sin(x)\right]x^2\right) \\
&= \left(2\cos(x) + (-\sin(x))(2x+3)\right) - \left(2x\sin(x) + \cos(x)\cdot x^2\right) \\
&= 2\cos(x) - (2x+3)\sin(x) - 2x\sin(x) + x^2\cos(x) \\
&= (x^2+2)\cos(x) - (4x+3)\sin(x) \quad \blacksquare
\end{aligned}$$

211. $\frac{d^4y}{dx^4}$ for $y = 5\cos(x)$

Sol: $\frac{dy}{dx} = -5\sin(x)$, $\quad \frac{d^2y}{dx^2} = -5\cos(x)$

$\frac{d^3y}{dx^3} = 5\sin(x)$, $\quad \frac{d^4y}{dx^4} = 5\cos(x)$ $\boxed{\diagup}$

212. $\frac{d^2y}{dx^2}$ for $y = \sec(x) + \cot(x)$

Sol: $\frac{dy}{dx} = \frac{d}{dx}\left[\sec(x) + \cot(x)\right] = \sec(x)\tan(x) - \csc^2(x)$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\sec(x)\tan(x) - \csc^2(x)\right]$$

$$= \frac{d}{dx}\left[\sec(x)\tan(x)\right] - \frac{d}{dx}\left[\csc(x)\cdot\csc(x)\right]$$

$$= \left(\frac{d}{dx}\left[\sec(x)\right]\tan(x) + \frac{d}{dx}\left[\tan(x)\right]\sec(x)\right) - \left(\frac{d}{dx}\left[\csc(x)\right]\csc(x) + \frac{d}{dx}\left[\csc(x)\right]\csc(x)\right)$$

$$= \sec(x)\tan(x)\cdot\tan(x) + \sec^2(x)\cdot\sec(x) - 2(-\csc(x)\cot(x))\csc(x)$$

$$= \sec(x)\tan^2(x) + \sec^3(x) + 2\csc^2(x)\cot(x)$$ $\boxed{\diagup}$

213. $\frac{d^3y}{dx^3}$ for $y = x^{10} - \sec(x)$

Sol: $\frac{dy}{dx} = \frac{d}{dx}\left[x^{10} - \sec(x)\right] = 10x^9 - \sec(x)\tan(x)$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[10x^9 - \sec(x)\tan(x)\right] = \frac{d}{dx}\left[10x^9\right] - \frac{d}{dx}\left[\sec(x)\tan(x)\right]$$

$$= 90x^8 - \left(\frac{d}{dx}\left[\sec(x)\right]\tan(x) + \sec(x)\frac{d}{dx}\left[\tan(x)\right]\right)$$

$$= 90x^8 - \left(\sec(x)\tan(x)\tan(x) + \sec(x)\cdot\sec^2(x)\right)$$

$$= 90x^8 - \sec(x)\left(\tan^2(x) + \sec^2(x)\right)$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left[90x^8 - \sec(x)\left(\tan^2(x) + \sec^2(x)\right)\right]$$

$$= \frac{d}{dx}\left[90x^8\right] - \frac{d}{dx}\left[\sec(x)\left(\tan^2(x) + \sec^2(x)\right)\right]$$

$$= 720x^7 - \left(\frac{d}{dx}\left[\sec(x)\right]\left(\tan^2(x) + \sec^2(x)\right) + \sec(x)\frac{d}{dx}\left[\tan^2(x) + \sec^2(x)\right]\right)$$

$$= 720x^7 - \left(\sec(x)\tan(x)\left(\tan^2(x) + \sec^2(x)\right)\right.$$
$$\left. + \sec(x)\left(2\tan(x)\sec^2(x) + 2\sec(x)\cdot\sec(x)\tan(x)\right)\right)$$

$$= 720x^7 - \sec(x)\left(\tan^3(x) + \sec^2(x)\tan(x) + 4\sec^2(x)\tan(x)\right)$$

$$= 720x^7 - \sec(x)\tan(x)\left(\tan^2(x) + 5\sec^2(x)\right) \quad \boxtimes$$

Note: It's possible your answer looks different but is still correct (if you used any trig identities, for example).