Derivative Rules

106–117. Calculate $\frac{df}{dx}$.

106. $f(x) = x^7 + 10$

Sol: $\frac{df}{dx} = \frac{d}{dx}[x^7+10] = \frac{d}{dx}[x^7] + \frac{d}{dx}[10] = 7x^{7-1} + 0 = 7x^6$

107. $f(x) = 5x^3 - x + 1$

Sol: $\frac{df}{dx} = \frac{d}{dx}[5x^3 - x + 1] = \frac{d}{dx}[5x^3] - \frac{d}{dx}[x] + \frac{d}{dx}[1]$

$= 5\frac{d}{dx}[x^3] - \frac{d}{dx}[x] + 0 = 5 \cdot 3x^2 - 1 = 15x^2 - 1$

108. $f(x) = 4x^2 - 7x$

Sol: $\frac{df}{dx} = \frac{d}{dx}[4x^2 - 7x] = \frac{d}{dx}[4x^2] - \frac{d}{dx}[7x]$

$= 4\frac{d}{dx}[x^2] - 7\frac{d}{dx}[x] = 4 \cdot 2x - 7 \cdot 1 = 8x - 7$

109. $f(x) = 8x^4 + 9x^2 - 1$

Sol: $\frac{df}{dx} = \frac{d}{dx}[8x^4 + 9x^2 - 1] = \frac{d}{dx}[8x^4] + \frac{d}{dx}[9x^2] - \frac{d}{dx}[1]$

$= 8\frac{d}{dx}[x^4] + 9\frac{d}{dx}[x^2] - 0 = 8 \cdot 4x^3 + 9 \cdot 2x = 32x^3 + 18x$

110. $f(x) = x^4 + \frac{2}{x}$

Sol: $\frac{df}{dx} = \frac{d}{dx}\left[x^4 + \frac{2}{x}\right] = \frac{d}{dx}\left[x^4 + 2x^{-1}\right]$

$= \frac{d}{dx}[x^4] + 2\frac{d}{dx}[x^{-1}] = 4x^3 + 2\cdot(-1)x^{-2} = 4x^3 - \frac{2}{x^2}$ $\blacksquare$

111. $f(x) = 3x\left(18x^4 + \frac{13}{x+1}\right)$

Sol 1: $\frac{df}{dx} = \frac{d}{dx}\left[3x\left(18x^4 + \frac{13}{x+1}\right)\right] = 3x\frac{d}{dx}\left[18x^4 + \frac{13}{x+1}\right] + \frac{d}{dx}[3x]\left(18x^4 + \frac{13}{x+1}\right)$

$= 3x\left(18\cdot 4x^3 - \frac{13}{(x+1)^2}\right) + 3\left(18x^4 + 13(x+1)^{-1}\right)$

$= 3x\left(72x^3 - \frac{13}{(x+1)^2}\right) + 3\left(18x^4 + \frac{13}{x+1}\right)$

$= 3\left(72x^4 - \frac{13x}{(x+1)^2} + 18x^4 + \frac{13(x+1)}{(x+1)^2}\right)$

$= 3\left(90x^4 + \frac{13}{(x+1)^2}\right) = 270x^4 + 39(x+1)^{-2}$ $\blacksquare$

Sol 2: $\frac{df}{dx} = \frac{d}{dx}\left[3x\left(18x^4 + \frac{13}{x+1}\right)\right] = \frac{d}{dx}\left[54x^5 + \frac{39x}{x+1}\right]$

$= \frac{d}{dx}[54x^5] + \frac{d}{dx}\left[\frac{39x}{x+1}\right] = 54\cdot 5x^4 + \frac{(x+1)\frac{d}{dx}[39x] - 39x\frac{d}{dx}[x+1]}{(x+1)^2}$

$= 270x^4 + \frac{39(x+1) - 39x}{(x+1)^2} = 270x^4 + \frac{39}{(x+1)^2}$ $\blacksquare$

112. $f(x) = (x+2)(2x^2-3)$

Sol 1: $\frac{df}{dx} = \frac{d}{dx}\left[(x+2)(2x^2-3)\right] = \frac{d}{dx}\left[2x^3 + 4x^2 - 3x - 6\right] = 6x^2 + 8x - 3$ $\blacksquare$

Sol 2: $\frac{df}{dx} = \frac{d}{dx}\left[(x+2)(2x^2-3)\right] = (x+2)\frac{d}{dx}[2x^2-3] + \frac{d}{dx}[x+2](2x^2-3)$

$= (x+2)\cdot 4x + 1\cdot(2x^2-3)$

$= 4x^2 + 8x + 2x^2 - 3 = 6x^2 + 8x - 3$ $\blacksquare$

113. $f(x) = x^2\left(\frac{2}{x^2} + \frac{5}{x^3}\right)$

Sol: $\frac{df}{dx} = \frac{d}{dx}\left[x^2\left(\frac{2}{x^2} + \frac{5}{x^3}\right)\right] = \frac{d}{dx}\left[2 + 5x^{-1}\right] = -5x^{-2}$ ▨

114. $f(x) = \frac{x^3+2x^2-4}{3}$

Sol: $\frac{df}{dx} = \frac{d}{dx}\left[\frac{x^3+2x^2-4}{3}\right] = \frac{1}{3}\frac{d}{dx}\left[x^3+2x^2-4\right] = \frac{1}{3}\left(3x^2+4x\right) = x^2+\frac{4}{3}x$ ▨

115. $f(x) = \frac{4x^3-2x+1}{x^2}$

Sol: $\frac{df}{dx} = \frac{d}{dx}\left[\frac{4x^3-2x+1}{x^2}\right] = \frac{d}{dx}\left[4x-2x^{-1}+x^{-2}\right]$

$= 4+2x^{-2}-2x^{-3}$ ▨

116. $f(x) = \frac{x^2+4}{x^2-4}$

Sol: $\frac{df}{dx} = \frac{d}{dx}\left[\frac{x^2+4}{x^2-4}\right] = \frac{(x^2-4)\frac{d}{dx}[x^2+4] - (x^2+4)\frac{d}{dx}[x^2-4]}{(x^2-4)^2}$

$= \frac{(x^2-4)(2x) - (x^2+4)(2x)}{(x^2-4)^2}$

$= \frac{2x(x^2-4-x^2-4)}{(x^2-4)^2} = \frac{-8x}{(x^2-4)^2}$ ▨

117. $f(x) = \frac{x+9}{x^2-7x+1}$

Sol: $\frac{df}{dx} = \frac{d}{dx}\left[\frac{x+9}{x^2-7x+1}\right] = \frac{(x^2-7x+1)\frac{d}{dx}[x+9] - (x+9)\frac{d}{dx}[x^2-7x+1]}{(x^2-7x+1)^2}$

$= \frac{(x^2-7x+1)\cdot 1 - (x+9)(2x-7)}{(x^2-7x+1)^2}$

$= \frac{x^2-7x+1-(2x^2-7x+18x-63)}{(x^2-7x+1)^2} = \frac{63-18x-x^2}{(x^2-7x+1)^2}$ ▨

118–121: Find an equation of the tangent line to $y=f(x)$ at P.

118. $y=3x^2+4x+1$, $P=(0,1)$

Sol: $\frac{dy}{dx}=\frac{d}{dx}[3x^2+4x+1] = 6x+4 = y'(x)$

$\therefore m = y'(0) = 6\cdot 0+4=4.$ $\therefore y-1=4(x-0)$ or $y=4x+1$ $\boxed{/\!/}$

119. $y=2\sqrt{x}+1$, $P=(4,5)$

Sol: $\frac{dy}{dx}=\frac{d}{dx}[2\sqrt{x}+1] = \frac{d}{dx}[2x^{1/2}+1] = x^{-1/2} = \frac{1}{\sqrt{x}} = y'(x)$

$\therefore m = y'(4) = \frac{1}{\sqrt{4}} = \frac{1}{2}$ $\therefore y-5=\frac{1}{2}(x-4)$ or $y=\frac{1}{2}x+3$ $\boxed{/\!/}$

120. $y=\frac{2x}{x-1}$, $P=(-1,1)$

Sol: $y'(x)=\frac{d}{dx}\left[\frac{2x}{x-1}\right] = \frac{(x-1)\frac{d}{dx}[2x]-2x\frac{d}{dx}[x-1]}{(x-1)^2} = \frac{(x-1)\cdot 2-2x\cdot 1}{(x-1)^2}$

$$= \frac{2x-2-2x}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

$\therefore m = y'(-1) = -\frac{2}{(-1-1)^2} = -\frac{2}{4} = -\frac{1}{2}$

$\therefore y-1 = -\frac{1}{2}(x-(-1))$ or $y=-\frac{1}{2}x+\frac{1}{2}$ $\boxed{/\!/}$

121. $y=\frac{2}{x}-\frac{3}{x^2}$, $P=(1,-1)$

Sol: $y'(x)=\frac{d}{dx}\left[\frac{2}{x}-\frac{3}{x^2}\right] = \frac{d}{dx}[2x^{-1}-3x^{-2}] = -2x^{-2}+6x^{-3}$

$\therefore m = y'(1) = -2\cdot 1^{-2}+6\cdot 1^{-3} = -2+6=4$

$\therefore y-(-1) = 4(x-1)$ or $y=4x-5$ $\boxed{/\!/}$

122–125: f and g are differentiable. Calculate $h'(x)$.

122. $h(x) = 4f(x) + \frac{g(x)}{7}$

Sol: $h'(x) = \frac{d}{dx}\left[4f(x) + \frac{1}{7}\cdot g(x)\right] = 4f'(x) + \frac{1}{7}\cdot g'(x)$ $\blacksquare$

123. $h(x) = x^3 f(x)$

Sol: $h'(x) = \frac{d}{dx}\left[x^3 f(x)\right] = \frac{d}{dx}\left[x^3\right] f(x) + x^3 \frac{d}{dx}\left[f(x)\right] = 3x^2 f(x) + x^3 f'(x)$ $\blacksquare$

124. $h(x) = \frac{f(x)g(x)}{2}$

Sol: $h'(x) = \frac{d}{dx}\left[\frac{1}{2}\cdot f(x)g(x)\right] = \frac{1}{2}\frac{d}{dx}\left[f(x)g(x)\right]$

$= \frac{1}{2}\left(f(x)g'(x) + g(x)f'(x)\right)$ $\blacksquare$

125. $h(x) = \frac{3f(x)}{g(x)+2}$

Sol: $h'(x) = \frac{(g(x)+2)\frac{d}{dx}\left[3f(x)\right] - 3f(x)\frac{d}{dx}\left[g(x)+2\right]}{(g(x))^2}$

$= \frac{3f'(x)(g(x)+2) - 3f(x)g'(x)}{(g(x))^2}$ $\blacksquare$

126 – 129: Use the table of values to calculate $h'(a)$.

x	1	2	3	4
$f(x)$	3	5	-2	0
$g(x)$	2	3	-4	6
$f'(x)$	-1	7	8	-3
$g'(x)$	4	1	2	9

126. $h(x) = x f(x) + 4g(x)$ at $a = 1$.

Sol: $h'(x) = \frac{d}{dx}\left[x f(x) + 4g(x)\right] = x f'(x) + f(x) + 4g'(x)$.

$\therefore h'(1) = 1 \cdot f'(1) + f(1) + 4g'(1) = 1 \cdot -1 + 3 + 4 \cdot 1 = 6$ $\boxed{}$

127. $h(x) = \frac{f(x)}{g(x)}$ at $a = 2$

Sol: $h'(x) = \frac{g(x) f'(x) - f(x) g'(x)}{(g(x))^2}$ $\therefore h'(2) = \frac{g(2) f'(2) - f(2) g'(2)}{(g(2))^2} = \frac{3 \cdot 7 - 5 \cdot 1}{3^2} = \frac{16}{9}$ $\boxed{}$

128. $h(x) = 2x + f(x) g(x)$ at $a = 3$

Sol: $h'(x) = 2 + f'(x) g(x) + f(x) g'(x)$

$\therefore h'(3) = 2 + 8 \cdot -4 + -2 \cdot 2 = -34$ $\boxed{}$

129. $h(x) = \frac{1}{x} + \frac{g(x)}{f(x)}$ at $a = 4$.

Sol: $h'(x) = \frac{d}{dx}\left[x^{-1} + \frac{g(x)}{f(x)}\right] = -x^{-2} + \frac{f(x) g'(x) - g(x) f'(x)}{(f(x))^2}$

HOWEVER: $f(4) = 0$, so $h(4)$ and $h'(4)$ DNE $\boxed{}$

133–139: Calculate the equation of the tangent line to f at a.

133. $f(x) = 2x^3 + 3x - x^2$ at $a = 2$

Sol: $f'(x) = 6x^2 + 3 - 2x$, $\quad f'(2) = 6\cdot 2^2 + 3 - 2\cdot 2 = 24 + 3 - 4 = 23.$

$y - y_0 = m(x - x_0)$ $\qquad f(2) = 2\cdot 2^3 + 3\cdot 2 - 2^2 = 16 + 6 - 4 = 18.$

$y - 18 = 23(x - 2)$ OR $y = 23x - 28.$ $\blacksquare$

134. $f(x) = \frac{1}{x} - x^2$ at $a = 1$

Sol: $f'(x) = -x^{-2} - 2x$ $\qquad f'(1) = -1^{-2} - 2\cdot 1 = -3$

$y - y_0 = m(x - x_0)$ $\qquad f(1) = \frac{1}{1} - 1^2 = 0$

$y - 0 = -3(x - 1)$ OR $y = -3x + 3$ $\blacksquare$

135. $f(x) = x^2 - x^{12} + 3x + 2$ at $a = 0$

Sol: $f'(x) = 2x - 12x^{11} + 3$ $\qquad f'(0) = 2\cdot 0 - 12\cdot 0^{11} + 3 = 3$

$y - y_0 = m(x - x_0)$ $\qquad f(0) = 0^2 - 0^{12} + 3\cdot 0 + 2 = 2$

$y - 2 = 3(x - 0)$ OR $y = 3x + 2$ $\blacksquare$

136. $f(x) = \frac{1}{x} - x^{2/3}$ at $a = -1$

Sol: $f'(x) = -x^{-2} - \frac{2}{3}x^{-1/3}$ $\qquad f'(-1) = -(-1)^{-2} - \frac{2}{3}(-1)^{-1/3} = -\frac{1}{3}$

$y - y_0 = m(x - x_0)$ $\qquad f(-1) = (-1)^{-1} - (-1)^{2/3} = -1 - 1 = -2$

$y - (-2) = -\frac{1}{3}(x - (-1))$ OR $y = -\frac{1}{3}x - \frac{7}{3}$ $\blacksquare$

137. $f(x) = 2x^3 + 4x^2 - 5x - 3$ at $a = -1$

Sol: $f'(x) = 6x^2 + 8x - 5$ $\qquad f'(-1) = 6(-1)^2 + 8(-1) - 5 = -7$

$y - y_0 = m(x - x_0)$ $\qquad f(-1) = 2(-1)^3 + 4(-1)^2 - 5(-1) - 3 = 4$

$y - 4 = -7(x - (-1))$ OR $y = -7x - 3$ $\blacksquare$

138. $f(x) = x^2 + \frac{4}{x} - 10$ at $a = 8$

Sol: $f'(x) = 2x - 4x^{-2} - 10$ $\qquad f'(8) = 2 \cdot 8 - 4 \cdot 8^{-2} - 8 = 6 - \frac{1}{16} = \frac{95}{16}$

$y - y_0 = m(x - x_0)$ $\qquad f(8) = 8^2 + \frac{4}{8} - 10 = 64 + \frac{1}{2} - 10 = \frac{109}{2}$

$y - \frac{109}{2} = \frac{95}{16}(x - 8)$ $\blacksquare$

139. $f(x) = (3x - x^2)(3 - x - x^2)$ at $a = 1$.

Sol: $f'(x) = \frac{d}{dx}\left[(3x - x^2)(3 - x - x^2)\right]$

$= \frac{d}{dx}\left[3x - x^2\right](3 - x - x^2) + (3x - x^2)\frac{d}{dx}\left[3 - x - x^2\right]$

$= (3 - 2x)(3 - x - x^2) + (3x - x^2)(-1 - 2x)$

$f'(1) = (3 - 2 \cdot 1)(3 - 1 - 1^2) + (3 \cdot 1 - 1^2)(-1 - 2 \cdot 1) = 1 \cdot 1 + 2 \cdot (-3) = -5$

$f(1) = (3 \cdot 1 - 1^2)(3 - 1 - 1^2) = 2 \cdot 1 = 2$

$y - 2 = -5(x - 1)$ OR $y = -5x + 7$ $\blacksquare$

140. Find the point on the graph of $f(x)=x^3$ such that the tangent line has an x-intercept of 6.

Sol: $f'(x)=3x^2$. We're looking for a line of the form

$$y-f(a)=f'(a)(x-a), \text{ i.e. } y=f'(a)(x-a)+f(a)$$

where $$0=f'(a)(6-a)+f(a).$$

Now we'll plug in what we have:

$$0=3a^2(6-a)+a^3=18a^2-2a^3=2a^2(9-a).$$

$$\Longleftrightarrow \quad a=0 \quad \text{or} \quad a=9$$

Check $a=0$: $f'(0)=3\cdot 0^2=0$, $f(0)=0^3=0$,

So the line $y-0=0(x-0)$ i.e. $y=0$ at $a=0$.

$\therefore$ the point $(0,0)$ works.

Check $a=9$: $f'(9)=3\cdot 9^2=243$, $f(9)=9^3=729$,

So the line $y-729=243(x-9)$, i.e. $y=243(x-9)+729$

at $x=6$, $y=243(6-9)+729=-729+729=0$.

$\therefore$ the point $(9,729)$ works.

141: Find the equation of the line passing through $(3,3)$ and tangent to $f(x) = \frac{6}{x-1}$.

Sol: $f'(x) = \frac{(x-1)\frac{d}{dx}[6] - \frac{d}{dx}[x-1]\cdot 6}{(x-1)^2} = \frac{(x-1)\cdot 0 - 1\cdot 6}{(x-1)^2} = \frac{-6}{(x-1)^2}$

So the general equation of the tangent line at a is

$y = f'(a)(x-a) + f(a)$ i.e. $y = \frac{-6}{(a-1)^2}(x-a) + \frac{6}{a-1}$

B/c $(3,3)$ is on this line, we know

$3 = -\frac{6}{(a-1)^2}(3-a) + \frac{6}{a-1}$ (multiply by $(a-1)^2$)

$\Longrightarrow 3(a-1)^2 = -6(3-a) + 6(a-1)$

$\Longleftrightarrow (a-1)^2 = -2(3-a) + 2(a-1)$

$\Longleftrightarrow a^2-2a+1 = -6+2a+2a-2$

$\Longleftrightarrow a^2-6a+9=0 \Longleftrightarrow (a-3)^2 = 0$

So $a=3$ is the only possible solution.

Check: $f(3) = \frac{6}{3-1} = 3$, $f'(3) = \frac{-6}{(3-1)^2} = -\frac{3}{2}$

So tangent line $y-3 = -\frac{3}{2}(x-3)$ (i.e. $y = -\frac{3}{2}x + \frac{15}{2}$)

and $3-3 = -\frac{3}{2}(3-3)$ is satisfied. ☑

142. Determine all points on the graph of $f(x) = x^3 + x^2 - x - 1$ for which (a) the tangent line is horizontal
(b) the tangent line has slope of -1.

Sol: $f'(x) = 3x^2 + 2x - 1$

(a) "horizontal" means "slope 0".

$$f'(x) = 0 \iff 3x^2 + 2x - 1 = 0 \iff (3x-1)(x+1) = 0$$
$$\iff 3x - 1 = 0 \quad \text{OR} \quad x + 1 = 0$$
$$\iff x = \tfrac{1}{3} \quad \text{OR} \quad x = -1$$

$\therefore$ points $\left(\tfrac{1}{3}, f(\tfrac{1}{3})\right) = \left(\tfrac{1}{3}, (\tfrac{1}{3})^3 + (\tfrac{1}{3})^2 - \tfrac{1}{3} - 1\right) = \left(\tfrac{1}{3}, \tfrac{1}{27} + \tfrac{3}{27} - \tfrac{9}{27} - \tfrac{27}{27}\right) = \left(\tfrac{1}{3}, \tfrac{-32}{27}\right)$

and $(-1, f(-1)) = (-1, (-1)^3 + (-1)^2 - (-1) - 1) = (-1, 0)$

(b) $f'(x) = -1 \iff 3x^2 - 2x - 1 = -1 \iff 3x^2 - 2x = 0$
$$\iff x(3x-2) = 0 \iff x = 0 \quad \text{OR} \quad 3x - 2 = 0$$
$$\iff x = 0 \quad \text{OR} \quad x = \tfrac{2}{3}$$

$\therefore$ points $(0, f(0)) = (0, 0^3 + 0^2 - 0 - 1) = (0, -1)$

and $\left(\tfrac{2}{3}, f(\tfrac{2}{3})\right) = \left(\tfrac{2}{3}, (\tfrac{2}{3})^3 + (\tfrac{2}{3})^2 - \tfrac{2}{3} - 1\right) = \left(\tfrac{2}{3}, \tfrac{8}{27} + \tfrac{4}{9} - \tfrac{2}{3} - 1\right) = \left(\tfrac{2}{3}, -\tfrac{25}{27}\right)$ $\boxed{\checkmark}$

143. Find a quadratic polynomial satisfying
$$f(1) = 5, \quad f'(1) = 3, \quad \text{and} \quad f''(1) = -6.$$

Sol: $f(x) = ax^2 + bx + c \quad \rightsquigarrow \quad 5 = f(1) = a + b + c$

$f'(x) = 2ax + b \quad \rightsquigarrow \quad 3 = f'(1) = 2a + b$

$f''(x) = 2a \quad \rightsquigarrow \quad -6 = f''(1) = 2a$

$$\therefore \begin{cases} a + b + c = 5 \\ 2a + b = 3 \\ 2a = -6 \end{cases} \rightsquigarrow \begin{cases} a = -3 \\ b = 3 - 2a = 3 - (-6) = 9 \\ c = 5 - a - b = 5 - (-3) - 9 = -1 \end{cases}$$

$\therefore f(x) = -3x^2 + 9x - 1$ $\boxed{\checkmark}$

144. A car driving along a freeway with traffic has traveled $s(t) = t^3 - 6t^2 + 9t$ meters in t seconds.

(a) When is the velocity of the car 0?

(b) When is the acceleration of the car 0?

Sol:

(a) $v(t) = s'(t) = 3t^2 - 12t + 9$

$$v(t) = 0 \Leftrightarrow 3t^2 - 12t + 9 = 0 \Leftrightarrow t^2 - 4t + 3 = 0 \Leftrightarrow (t-1)(t-3) = 0$$
$$\Leftrightarrow t = 1 \quad \text{or} \quad t = 3.$$

$\therefore$ the car is stopped at 1 and 3 seconds.

(b) $a(t) = v'(t) = 6t - 12$

$$a(t) = 0 \Leftrightarrow 6t - 12 = 0 \Leftrightarrow t = 2$$

$\therefore$ the car has acceleration 0 at 2 seconds. $\boxed{//}$

145. A herring swimming along a straight line has travelled $s(t) = \frac{t^2}{t^2+2}$ feet in t seconds. Determine the velocity of the herring when it has traveled 3 seconds.

Sol: $$v(t) = s'(t) = \frac{d}{dt}\left[\frac{t^2}{t^2+2}\right] = \frac{(t^2+2)\frac{d}{dt}[t^2] - t^2\frac{d}{dt}[t^2+2]}{(t^2+2)^2}$$

$$= \frac{(t^2+2)\cdot 2t - t^2 \cdot 2t}{(t^2+2)^2} = \frac{4t}{(t^2+2)^2}$$

$$\therefore v(3) = \frac{4 \cdot 3}{(3^2+2)^2} = \frac{12}{11^2} = \frac{12}{121}$$

$\therefore$ the velocity at 3 seconds is $\frac{12}{121}$ ft/s. $\boxed{//}$

146. The population in millions of arctic flounder in the atlantic ocean is modeled by the function $P(t) = \dfrac{8t+3}{.2t^2+1}$ where t is measured in years. (A) Determine the initial flounder population. (B) Determine $P'(10)$ and briefly interpret the result.

Sol: (A) $P(0) = \dfrac{8\cdot 0+3}{.2\cdot 0^2+1} = 3$. $\therefore$ the initial population is 3 million flounder.

(B) $$P'(t) = \frac{d}{dt}\left[\frac{8t+3}{.2t^2+1}\right] = \frac{(.2t^2+1)\frac{d}{dt}[8t+3] - \frac{d}{dt}[.2t^2+1](8t+3)}{(.2t^2+1)^2}$$

$$= \frac{(.2t^2+1)\cdot 8 - .4t\,(8t+3)}{(.2t^2+1)^2}$$

$$\therefore P'(10) = \frac{(.2\cdot 10^2+1)\cdot 8 - .4\cdot 10\,(8\cdot 10+3)}{(.2\cdot 10^2+1)^2}$$

$$= \frac{21\cdot 8 - 4\cdot 83}{21^2} = \frac{168-332}{441} = -\frac{164}{441}$$

$\therefore$ at 10 years into the model the flounder population is decreasing at a rate of $\frac{164}{441}$ millions per year. $\boxed{\text{☺}}$

147. The concentration of antibiotic in the bloodstream t hours after injection is given by the function $C(t) = \frac{2t^2+t}{t^3+50}$ where C is measured in milligrams per liter of blood.

(A) Find the rate of change of $C(t)$.

(B) Determine the rate of change at $t = 8, 12, 24$, and 36.

(C) Describe the trend as time progresses based on these four data points (from B).

Sol: (A) $C'(t) = \frac{d}{dt}\left[\frac{2t^2+t}{t^3+50}\right] = \frac{(t^3+50)\frac{d}{dt}[2t^2+t] - (2t^2+t)\frac{d}{dt}[t^3+50]}{(t^3+50)^2}$

$$= \frac{(t^3+50)(4t+1) - (2t^2+t)(3t^2)}{(t^3+50)^2}$$

(B)

t	$C'(t)$
8	-13.46
12	-23.89
24	-49.47
36	-73.77

← rounded to 2 decimal places

I used the calculator on my computer to get these approximations

(C) Over time, the concentration decreases more rapidly. $\boxed{\checkmark}$

148. A book publisher has a cost function given by $C(n) = \frac{n^3+2n+3}{n^2}$ where n is the number of copies of a book in thousands and C is the cost per book measured in dollars. Evaluate $C'(2)$ and explain its meaning.

Sol: $$C'(n) = \frac{d}{dn}\left[\frac{n^3+2n+3}{n^2}\right] = \frac{n^2\frac{d}{dn}[n^3+2n+3] - (n^3+2n+3)\frac{d}{dn}[n^2]}{(n^2)^2}$$

$$= \frac{n^2\cdot(3n^2+2) - (n^3+2n+3)\cdot 2n}{n^4}$$

$$= \frac{n(3n^3+2n - 2n^3-4n-6)}{n\cdot n^3}$$

$$= \frac{n^3-2n-6}{n^3}$$

$$\therefore C'(2) = \frac{2^3-2\cdot2-6}{2^3} = \frac{8-4-6}{8} = -\frac{2}{8} = -\frac{1}{4}$$

$C'(n)$ is the (instantaneous) change in cost. Thus $C'(2) = -\frac{1}{4}$ means the (instantaneous) change in cost of producing 2000 copies of a book is decreasing at a rate of \$.25 per additional copy. $\boxed{/\!/}$

149. According to Newton's Law of universal gravitation, the force F between two bodies of constant mass m_1 and m_2 is $F = \frac{Gm_1m_2}{r^2}$ where G is the gravitation constant and r is the distance between the bodies.

Ⓐ Find the rate of change of force with respect to distance.

Ⓑ Find the rate of change of gravitational force acting on two bodies of mass 1000 kg when they are 10 meters apart (assume $G = 6.67 \cdot 10^{-11}\ Nm^2/kg^2$).

Ⓐ $\frac{dF}{dr} = \frac{d}{dr}\left[\frac{Gm_1m_2}{r^2}\right] = \frac{d}{dr}\left[Gm_1m_2r^{-2}\right] = -2Gm_1m_2r^{-3}$

Ⓑ $F'(10) = -2 \cdot 6.67 \cdot 10^{-11}\ Nm^2/kg^2 \cdot 10^3 kg \cdot 10^3 kg \cdot (10m)^{-3}$

$= -2 \cdot 6.67 \cdot 10^{-11+3+3-3} \left(\frac{Nm^2 kg^2}{kg^2 m^3}\right)$

$\begin{array}{r} 6.67 \\ \times\ 2 \\ \hline 12.34 \end{array}$

$= -12.34 \cdot 10^{-8}\ N/m = -1.234 \cdot 10^{-7}\ N/m$

$\therefore$ the gravitational force is decreasing at a rate of $1.234 \cdot 10^{-7}$ N/m ☒