Derivative Rules

106–117. Calculate $\frac{df}{dx}$.

106. $f(x) = x^7 + 10$

Sol: $\frac{df}{dx} = \frac{d}{dx}[x^7+10] = \frac{d}{dx}[x^7] + \frac{d}{dx}[10] = 7x^{7-1} + 0 = 7x^6$ $\blacksquare$

107. $f(x) = 5x^3 - x + 1$

Sol: $\frac{df}{dx} = \frac{d}{dx}\left[5x^3 - x + 1\right] = \frac{d}{dx}\left[5x^3\right] - \frac{d}{dx}[x] + \frac{d}{dx}[1]$

$= 5\frac{d}{dx}[x^3] - \frac{d}{dx}[x] + 0 = 5 \cdot 3x^2 - 1 = 15x^2 - 1$ $\blacksquare$

108. $f(x) = 4x^2 - 7x$

Sol: $\frac{df}{dx} = \frac{d}{dx}[4x^2 - 7x] = \frac{d}{dx}[4x^2] - \frac{d}{dx}[7x]$

$= 4\frac{d}{dx}[x^2] - 7\frac{d}{dx}[x] = 4 \cdot 2x - 7 \cdot 1 = 8x - 7$ $\blacksquare$

109. $f(x) = 8x^4 + 9x^2 - 1$

Sol: $\frac{df}{dx} = \frac{d}{dx}\left[8x^4 + 9x^2 - 1\right] = \frac{d}{dx}\left[8x^4\right] + \frac{d}{dx}[9x^2] - \frac{d}{dx}[1]$

$= 8\frac{d}{dx}[x^4] + 9\frac{d}{dx}[x^2] - 0 = 8 \cdot 4x^3 + 9 \cdot 2x = 32x^3 + 18x$ $\blacksquare$

110. $f(x) = x^4 + \frac{2}{x}$

Sol: $\frac{df}{dx} = \frac{d}{dx}\left[x^4 + \frac{2}{x}\right] = \frac{d}{dx}\left[x^4 + 2x^{-1}\right]$

$= \frac{d}{dx}[x^4] + 2\frac{d}{dx}[x^{-1}] = 4x^3 + 2\cdot(-1)x^{-2} = 4x^3 - \frac{2}{x^2}$ $\blacksquare$

111. $f(x) = 3x\left(18x^4 + \frac{13}{x+1}\right)$

Sol 1: $\frac{df}{dx} = \frac{d}{dx}\left[3x\left(18x^4 + \frac{13}{x+1}\right)\right] = 3x\frac{d}{dx}\left[18x^4 + \frac{13}{x+1}\right] + \frac{d}{dx}[3x]\left(18x^4 + \frac{13}{x+1}\right)$

$= 3x\left(18\cdot 4x^3 - \frac{13}{(x+1)^2}\right) + 3\left(18x^4 + 13(x+1)^{-1}\right)$

$= 3x\left(72x^3 - \frac{13}{(x+1)^2}\right) + 3\left(18x^4 + \frac{13}{x+1}\right)$

$= 3\left(72x^4 - \frac{13x}{(x+1)^2} + 18x^4 + \frac{13(x+1)}{(x+1)^2}\right)$

$= 3\left(90x^4 + \frac{13}{(x+1)^2}\right) = 270x^4 + 39(x+1)^{-2}$ $\blacksquare$

Sol 2: $\frac{df}{dx} = \frac{d}{dx}\left[3x\left(18x^4 + \frac{13}{x+1}\right)\right] = \frac{d}{dx}\left[54x^5 + \frac{39x}{x+1}\right]$

$= \frac{d}{dx}[54x^5] + \frac{d}{dx}\left[\frac{39x}{x+1}\right] = 54\cdot 5x^4 + \frac{(x+1)\frac{d}{dx}[39x] - 39x\frac{d}{dx}[x+1]}{(x+1)^2}$

$= 270x^4 + \frac{39(x+1) - 39x}{(x+1)^2} = 270x^4 + \frac{39}{(x+1)^2}$ $\blacksquare$

112. $f(x) = (x+2)(2x^2-3)$

Sol 1: $\frac{df}{dx} = \frac{d}{dx}\left[(x+2)(2x^2-3)\right] = \frac{d}{dx}\left[2x^3 + 4x^2 - 3x - 6\right] = 6x^2 + 8x - 3$ $\blacksquare$

Sol 2: $\frac{df}{dx} = \frac{d}{dx}\left[(x+2)(2x^2-3)\right] = (x+2)\frac{d}{dx}[2x^2-3] + \frac{d}{dx}[x+2](2x^2-3)$

$= (x+2)\cdot 4x + 1\cdot(2x^2-3)$

$= 4x^2 + 8x + 2x^2 - 3 = 6x^2 + 8x - 3$ $\blacksquare$

113. $f(x) = x^2\left(\frac{2}{x^2} + \frac{5}{x^3}\right)$

Sol: $\frac{df}{dx} = \frac{d}{dx}\left[x^2\left(\frac{2}{x^2} + \frac{5}{x^3}\right)\right] = \frac{d}{dx}\left[2 + 5x^{-1}\right] = -5x^{-2}$ $\boxed{//}$

114. $f(x) = \frac{x^3+2x^2-4}{3}$

Sol: $\frac{df}{dx} = \frac{d}{dx}\left[\frac{x^3+2x^2-4}{3}\right] = \frac{1}{3}\frac{d}{dx}\left[x^3+2x^2-4\right] = \frac{1}{3}\left(3x^2+4x\right) = x^2+\frac{4}{3}x$ $\boxed{//}$

115. $f(x) = \frac{4x^3-2x+1}{x^2}$

Sol: $\frac{df}{dx} = \frac{d}{dx}\left[\frac{4x^3-2x+1}{x^2}\right] = \frac{d}{dx}\left[4x - 2x^{-1} + x^{-2}\right]$

$= 4 + 2x^{-2} - 2x^{-3}$ $\boxed{//}$

116. $f(x) = \frac{x^2+4}{x^2-4}$

Sol: $\frac{df}{dx} = \frac{d}{dx}\left[\frac{x^2+4}{x^2-4}\right] = \frac{(x^2-4)\frac{d}{dx}[x^2+4] - (x^2+4)\frac{d}{dx}[x^2-4]}{(x^2-4)^2}$

$= \frac{(x^2-4)(2x) - (x^2+4)(2x)}{(x^2-4)^2}$

$= \frac{2x(x^2-4-x^2-4)}{(x^2-4)^2} = \frac{-8x}{(x^2-4)^2}$ $\boxed{//}$

117. $f(x) = \frac{x+9}{x^2-7x+1}$

Sol: $\frac{df}{dx} = \frac{d}{dx}\left[\frac{x+9}{x^2-7x+1}\right] = \frac{(x^2-7x+1)\frac{d}{dx}[x+9] - (x+9)\frac{d}{dx}[x^2-7x+1]}{(x^2-7x+1)^2}$

$= \frac{(x^2-7x+1)\cdot 1 - (x+9)(2x-7)}{(x^2-7x+1)^2}$

$= \frac{x^2-7x+1-(2x^2-7x+18x-63)}{(x^2-7x+1)^2} = \frac{63-18x-x^2}{(x^2-7x+1)^2}$ $\boxed{//}$

118–121: Find an equation of the tangent line to $y=f(x)$ at P.

118. $y=3x^2+4x+1$, $P=(0,1)$

Sol: $\frac{dy}{dx}=\frac{d}{dx}[3x^2+4x+1]=6x+4=y'(x)$

$\therefore m=y'(0)=6\cdot 0+4=4$. $\therefore y-1=4(x-0)$ or $y=4x+1$ $\boxed{}$

119. $y=2\sqrt{x}+1$, $P=(4,5)$

Sol: $\frac{dy}{dx}=\frac{d}{dx}[2\sqrt{x}+1]=\frac{d}{dx}[2x^{1/2}+1]=x^{-1/2}=\frac{1}{\sqrt{x}}=y'(x)$

$\therefore m=y'(4)=\frac{1}{\sqrt{4}}=\frac{1}{2}$ $\therefore y-5=\frac{1}{2}(x-4)$ or $y=\frac{1}{2}x+3$ $\boxed{}$

120. $y=\frac{2x}{x-1}$, $P=(-1,1)$

Sol: $y'(x)=\frac{d}{dx}\left[\frac{2x}{x-1}\right]=\frac{(x-1)\frac{d}{dx}[2x]-2x\frac{d}{dx}[x-1]}{(x-1)^2}=\frac{(x-1)\cdot 2-2x\cdot 1}{(x-1)^2}$

$$=\frac{2x-2-2x}{(x-1)^2}=\frac{-2}{(x-1)^2}$$

$\therefore m=y'(-1)=-\frac{2}{(-1-1)^2}=-\frac{2}{4}=-\frac{1}{2}$

$\therefore y-1=-\frac{1}{2}(x-(-1))$ or $y=-\frac{1}{2}x+\frac{1}{2}$ $\boxed{}$

121. $y=\frac{2}{x}-\frac{3}{x^2}$, $P=(1,-1)$

Sol: $y'(x)=\frac{d}{dx}\left[\frac{2}{x}-\frac{3}{x^2}\right]=\frac{d}{dx}[2x^{-1}-3x^{-2}]=-2x^{-2}+6x^{-3}$

$\therefore m=y'(1)=-2\cdot 1^{-2}+6\cdot 1^{-3}=-2+6=4$

$\therefore y-(-1)=4(x-1)$ or $y=4x-5$ $\boxed{}$

122–125: f and g are differentiable. Calculate $h'(x)$.

122. $h(x) = 4f(x) + \frac{g(x)}{7}$

Sol: $h'(x) = \frac{d}{dx}\left[4f(x) + \frac{1}{7}\cdot g(x)\right] = 4f'(x) + \frac{1}{7}\cdot g'(x)$ $\boxtimes$

123. $h(x) = x^3 f(x)$

Sol: $h'(x) = \frac{d}{dx}\left[x^3 f(x)\right] = \frac{d}{dx}[x^3] f(x) + x^3 \frac{d}{dx}[f(x)] = 3x^2 f(x) + x^3 f'(x)$ $\boxtimes$

124. $h(x) = \frac{f(x)g(x)}{2}$

Sol: $h'(x) = \frac{d}{dx}\left[\frac{1}{2}\cdot f(x)g(x)\right] = \frac{1}{2}\frac{d}{dx}\left[f(x)g(x)\right]$

$= \frac{1}{2}\left(f(x)g'(x) + g(x)f'(x)\right)$ $\boxtimes$

125. $h(x) = \frac{3f(x)}{g(x)+2}$

Sol: $h'(x) = \frac{(g(x)+2)\frac{d}{dx}[3f(x)] - 3f(x)\frac{d}{dx}[g(x)+2]}{(g(x))^2}$

$= \frac{3f'(x)(g(x)+2) - 3f(x)g'(x)}{(g(x))^2}$ $\boxtimes$

126–129: Use the table of values to calculate $h'(a)$.

x	1	2	3	4
$f(x)$	3	5	-2	0
$g(x)$	2	3	-4	6
$f'(x)$	-1	7	8	-3
$g'(x)$	4	1	2	9

126. $h(x) = xf(x) + 4g(x)$ at $a = 1$.

Sol: $h'(x) = \frac{d}{dx}\left[xf(x) + 4g(x)\right] = xf'(x) + f(x) + 4g'(x)$.

$\therefore h'(1) = 1 \cdot f'(1) + f(1) + 4g'(1) = 1 \cdot -1 + 3 + 4 \cdot 1 = 6$ ☒

127. $h(x) = \frac{f(x)}{g(x)}$ at $a = 2$

Sol: $h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$ $\therefore h'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2} = \frac{3 \cdot 7 - 5 \cdot 1}{3^2} = \frac{16}{9}$ ☒

128. $h(x) = 2x + f(x)g(x)$ at $a = 3$

Sol: $h'(x) = 2 + f'(x)g(x) + f(x)g'(x)$

$\therefore h'(3) = 2 + 8 \cdot -4 + -2 \cdot 2 = -34$ ☒

129. $h(x) = \frac{1}{x} + \frac{g(x)}{f(x)}$ at $a = 4$.

Sol: $h'(x) = \frac{d}{dx}\left[x^{-1} + \frac{g(x)}{f(x)}\right] = -x^{-2} + \frac{f(x)g'(x) - g(x)f'(x)}{(f(x))^2}$

HOWEVER: $f(4) = 0$, so $h(4)$ and $h'(4)$ DNE ☒

133–139. Calculate the equation of the tangent line to f at a.

133. $f(x) = 2x^3 + 3x - x^2$ at $a = 2$

Sol: $f'(x) = 6x^2 + 3 - 2x$, $\quad f'(2) = 6\cdot 2^2 + 3 - 2\cdot 2 = 24 + 3 - 4 = 23.$

$y - y_0 = m(x - x_0)$ $\quad f(2) = 2\cdot 2^3 + 3\cdot 2 - 2^2 = 16 + 6 - 4 = 18.$

$y - 18 = 23(x - 2)$ OR $y = 23x - 28.$ $\blacksquare$

134. $f(x) = \frac{1}{x} - x^2$ at $a = 1$

Sol: $f'(x) = -x^{-2} - 2x$ $\quad f'(1) = -1^{-2} - 2\cdot 1 = -3$

$y - y_0 = m(x - x_0)$ $\quad f(1) = \frac{1}{1} - 1^2 = 0$

$y - 0 = -3(x - 1)$ OR $y = -3x + 3$ $\blacksquare$

135. $f(x) = x^2 - x^{12} + 3x + 2$ at $a = 0$

Sol: $f'(x) = 2x - 12x^{11} + 3$ $\quad f'(0) = 2\cdot 0 - 12\cdot 0^{11} + 3 = 3$

$y - y_0 = m(x - x_0)$ $\quad f(0) = 0^2 - 0^{12} + 3\cdot 0 + 2 = 2$

$y - 2 = 3(x - 0)$ OR $y = 3x + 2$ $\blacksquare$

136. $f(x) = \frac{1}{x} - x^{2/3}$ at $a = -1$

Sol: $f'(x) = -x^{-2} - \frac{2}{3}x^{-1/3}$ $\quad f'(-1) = -(-1)^{-2} - \frac{2}{3}(-1)^{-1/3} = -\frac{1}{3}$

$y - y_0 = m(x - x_0)$ $\quad f(-1) = (-1)^{-1} - (-1)^{2/3} = -1 - 1 = -2$

$y - (-2) = -\frac{1}{3}(x - (-1))$ OR $y = -\frac{1}{3}x - \frac{7}{3}$ $\blacksquare$

137. $f(x) = 2x^3 + 4x^2 - 5x - 3$ at $a = -1$

Sol: $f'(x) = 6x^2 + 8x - 5$ $\quad f'(-1) = 6(-1)^2 + 8(-1) - 5 = -7$

$y - y_0 = m(x - x_0)$ $\quad f(-1) = 2(-1)^3 + 4(-1)^2 - 5(-1) - 3 = 4$

$y - 4 = -7(x - (-1))$ OR $y = -7x - 3$ $\boxed{/\!/}$

138. $f(x) = x^2 + \frac{4}{x} - 10$ at $a = 8$

Sol: $f'(x) = 2x - 4x^{-2} - 10$ $\quad f'(8) = 2 \cdot 8 - 4 \cdot 8^{-2} - 8 = 6 - \frac{1}{16} = \frac{95}{16}$

$y - y_0 = m(x - x_0)$ $\quad f(8) = 8^2 + \frac{4}{8} - 10 = 64 + \frac{1}{2} - 10 = \frac{109}{2}$

$y - \frac{109}{2} = \frac{95}{16}(x - 8)$ $\boxed{/\!/}$

139. $f(x) = (3x - x^2)(3 - x - x^2)$ at $a = 1$.

Sol: $f'(x) = \frac{d}{dx}\left[(3x - x^2)(3 - x - x^2)\right]$

$= \frac{d}{dx}[3x - x^2](3 - x - x^2) + (3x - x^2)\frac{d}{dx}[3 - x - x^2]$

$= (3 - 2x)(3 - x - x^2) + (3x - x^2)(-1 - 2x)$

$f'(1) = (3 - 2 \cdot 1)(3 - 1 - 1^2) + (3 \cdot 1 - 1^2)(-1 - 2 \cdot 1) = 1 \cdot 1 + 2 \cdot (-3) = -5$

$f(1) = (3 \cdot 1 - 1^2)(3 - 1 - 1^2) = 2 \cdot 1 = 2$

$y - 2 = -5(x - 1)$ OR $y = -5x + 7$ $\boxed{/\!/}$

140. Find the point on the graph of $f(x)=x^3$ such that the tangent line has an x-intercept of 6.

Sol: $f'(x)=3x^2$. We're looking for a line of the form

$y-f(a)=f'(a)(x-a)$, i.e. $y=f'(a)(x-a)+f(a)$

where $0=f'(a)(6-a)+f(a)$.

Now we'll plug in what we have:

$$0=3a^2(6-a)+a^3=18a^2-2a^3=2a^2(9-a).$$

$\Longleftrightarrow$ $a=0$ or $a=9$

Check $a=0$: $f'(0)=3\cdot 0^2=0$, $f(0)=0^3=0$,

so the line $y-0=0(x-0)$ i.e. $y=0$ at $a=0$.

$\therefore$ the point $(0,0)$ works.

Check $a=9$: $f'(9)=3\cdot 9^2=243$, $f(9)=9^3=729$,

So the line $y-729=243(x-9)$, i.e. $y=243(x-9)+729$

at $x=6$, $y=243(6-9)+729=-729+729=0$.

$\therefore$ the point $(9,729)$ works.

141: Find the equation of the line passing through $(3,3)$ and tangent to $f(x)=\frac{6}{x-1}$.

Sol: $f'(x) = \frac{(x-1)\frac{d}{dx}[6]-\frac{d}{dx}[x-1]\cdot 6}{(x-1)^2} = \frac{(x-1)\cdot 0 - 1\cdot 6}{(x-1)^2} = \frac{-6}{(x-1)^2}$

So the general equation of the tangent line at a is

$$y = f'(a)(x-a)+f(a) \quad \text{i.e.} \quad y=\frac{-6}{(a-1)^2}(x-a)+\frac{6}{a-1}$$

B/c $(3,3)$ is on this line, we know

$$3 = -\frac{6}{(a-1)^2}(3-a)+\frac{6}{a-1} \qquad (\text{multiply by } (a-1)^2)$$

$$\Longrightarrow 3(a-1)^2 = -6(3-a)+6(a-1)$$

$$\Longleftrightarrow (a-1)^2 = -2(3-a)+2(a-1)$$

$$\Longleftrightarrow a^2-2a+1 = -6+2a+2a-2$$

$$\Longleftrightarrow a^2-6a+9=0 \Longleftrightarrow (a-3)^2=0$$

So $a=3$ is the only possible solution.

Check: $f(3)=\frac{6}{3-1}=3$, $f'(3)=\frac{-6}{(3-1)^2}=-\frac{3}{2}$

So tangent line $y-3=-\frac{3}{2}(x-3)$ (i.e. $y=-\frac{3}{2}x+\frac{15}{2}$)

and $3-3=-\frac{3}{2}(3-3)$ is satisfied. ☒

142. Determine all points on the graph of $f(x) = x^3 + x^2 - x - 1$ for which (a) the tangent line is horizontal (b) the tangent line has slope of -1.

Sol: $f'(x) = 3x^2 + 2x - 1$

(a) "horizontal" means "slope 0".

$$f'(x) = 0 \iff 3x^2 + 2x - 1 = 0 \iff (3x-1)(x+1) = 0$$

$$\iff 3x - 1 = 0 \quad \text{OR} \quad x + 1 = 0$$

$$\iff x = \frac{1}{3} \quad \text{OR} \quad x = -1$$

$\therefore$ points $\left(\frac{1}{3}, f\left(\frac{1}{3}\right)\right) = \left(\frac{1}{3}, \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^2 - \frac{1}{3} - 1\right) = \left(\frac{1}{3}, \frac{1}{27} + \frac{3}{27} - \frac{9}{27} - \frac{27}{27}\right) = \left(\frac{1}{3}, -\frac{32}{27}\right)$

and $(-1, f(-1)) = (-1, (-1)^3 + (-1)^2 - (-1) - 1) = (-1, 0)$

(b) $f'(x) = -1 \iff 3x^2 - 2x - 1 = -1 \iff 3x^2 - 2x = 0$

$$\iff x(3x-2) = 0 \iff x = 0 \quad \text{OR} \quad 3x - 2 = 0$$

$$\iff x = 0 \quad \text{OR} \quad x = \frac{2}{3}$$

$\therefore$ points $(0, f(0)) = (0, 0^3 + 0^2 - 0 - 1) = (0, -1)$

and $\left(\frac{2}{3}, f\left(\frac{2}{3}\right)\right) = \left(\frac{2}{3}, \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^2 - \frac{2}{3} - 1\right) = \left(\frac{2}{3}, \frac{8}{27} + \frac{4}{9} - \frac{2}{3} - 1\right) = \left(\frac{2}{3}, -\frac{25}{27}\right)$ $\boxed{\checkmark}$

143. Find a quadratic polynomial satisfying $f(1) = 5$, $f'(1) = 3$, and $f''(1) = -6$.

Sol: $f(x) = ax^2 + bx + c \rightsquigarrow 5 = f(1) = a + b + c$

$f'(x) = 2ax + b \rightsquigarrow 3 = f'(1) = 2a + b$

$f''(x) = 2a \rightsquigarrow -6 = f''(1) = 2a$

$$\therefore \begin{cases} a + b + c = 5 \\ 2a + b = 3 \\ 2a = -6 \end{cases} \rightsquigarrow \begin{cases} a = -3 \\ b = 3 - 2a = 3 - (-6) = 9 \\ c = 5 - a - b = 5 - (-3) - 9 = -1 \end{cases}$$

$\therefore f(x) = -3x^2 + 9x - 1$ $\boxed{\checkmark}$

144. A car driving along a freeway with traffic has traveled $s(t) = t^3 - 6t^2 + 9t$ meters in t seconds.

ⓐ When is the velocity of the car 0?

ⓑ When is the acceleration of the car 0?

Sol:

ⓐ $v(t) = s'(t) = 3t^2 - 12t + 9$

$$v(t) = 0 \Leftrightarrow 3t^2 - 12t + 9 = 0 \Leftrightarrow t^2 - 4t + 3 = 0 \Leftrightarrow (t-1)(t-3) = 0$$
$$\Leftrightarrow t = 1 \quad \text{or} \quad t = 3.$$

$\therefore$ the car is stopped at 1 and 3 seconds.

ⓑ $a(t) = v'(t) = 6t - 12$

$$a(t) = 0 \Leftrightarrow 6t - 12 = 0 \Leftrightarrow t = 2$$

$\therefore$ the car has acceleration 0 at 2 seconds. ☒

145. A herring swimming along a straight line has travelled $s(t) = \frac{t^2}{t^2+2}$ feet in t seconds. Determine the velocity of the herring when it has traveled 3 seconds.

Sol:
$$v(t) = s'(t) = \frac{d}{dt}\left[\frac{t^2}{t^2+2}\right] = \frac{(t^2+2)\frac{d}{dt}[t^2] - t^2\frac{d}{dt}[t^2+2]}{(t^2+2)^2}$$

$$= \frac{(t^2+2)\cdot 2t - t^2 \cdot 2t}{(t^2+2)^2} = \frac{4t}{(t^2+2)^2}$$

$$\therefore v(3) = \frac{4\cdot 3}{(3^2+2)^2} = \frac{12}{11^2} = \frac{12}{121}$$

$\therefore$ the velocity at 3 seconds is $\frac{12}{121}$ ft/s. ☒

146. The population in millions of arctic flounder in the atlantic ocean is modeled by the function $P(t) = \dfrac{8t+3}{.2t^2+1}$ where t is measured in years. (A) Determine the initial flounder population. (B) Determine $P'(10)$ and briefly interpret the result.

Sol: (A) $P(0) = \dfrac{8\cdot 0+3}{.2\cdot 0^2+1} = 3$. $\therefore$ the initial population is 3 million flounder.

(B) $$P'(t) = \frac{d}{dt}\left[\frac{8t+3}{.2t^2+1}\right] = \frac{(.2t^2+1)\frac{d}{dt}[8t+3] - \frac{d}{dt}[.2t^2+1](8t+3)}{(.2t^2+1)^2}$$

$$= \frac{(.2t^2+1)\cdot 8 - .4t\,(8t+3)}{(.2t^2+1)^2}$$

$$\therefore P'(10) = \frac{(.2\cdot 10^2+1)\cdot 8 - .4\cdot 10\,(8\cdot 10+3)}{(.2\cdot 10^2+1)^2}$$

$$= \frac{21\cdot 8 - 4\cdot 83}{21^2} = \frac{168 - 332}{441} = -\frac{164}{441}$$

$\therefore$ at 10 years into the model the flounder population is decreasing at a rate of $\frac{164}{441}$ millions per year. $\boxed{\text{↯}}$

147. The concentration of antibiotic in the bloodstream t hours after injection is given by the function $C(t) = \frac{2t^2+t}{t^3+50}$ where C is measured in milligrams per liter of blood.

(A) Find the rate of change of $C(t)$.

(B) Determine the rate of change at $t = 8, 12, 24$, and 36.

(C) Describe the trend as time progresses based on these four data points (from B).

Sol: (A) $C'(t) = \frac{d}{dt}\left[\frac{2t^2+t}{t^3+50}\right] = \frac{(t^3+50)\frac{d}{dt}[2t^2+t] - (2t^2+t)\frac{d}{dt}[t^3+50]}{(t^3+50)^2}$

$$= \frac{(t^3+50)(4t+1) - (2t^2+t)(3t^2)}{(t^3+50)^2}$$

(B)

t	$C'(t)$
8	-13.46
12	-23.89
24	-49.47
36	-73.77

← rounded to 2 decimal places

I used the calculator on my computer to get these approximations

(C) Over time, the concentration decreases more rapidly. $\boxed{\checkmark}$

148. A book publisher has a cost function given by

$$C(n) = \frac{n^3 + 2n + 3}{n^2}$$

where n is the number of copies of a book in thousands and C is the cost per book measured in dollars. Evaluate $C'(2)$ and explain its meaning.

Sol:

$$C'(n) = \frac{d}{dn}\left[\frac{n^3+2n+3}{n^2}\right] = \frac{n^2 \frac{d}{dn}[n^3+2n+3] - (n^3+2n+3)\frac{d}{dn}[n^2]}{(n^2)^2}$$

$$= \frac{n^2 \cdot (3n^2+2) - (n^3+2n+3)\cdot 2n}{n^4}$$

$$= \frac{n(3n^3+2n - 2n^3 - 4n - 6)}{n \cdot n^3}$$

$$= \frac{n^3 - 2n - 6}{n^3}$$

$$\therefore C'(2) = \frac{2^3 - 2\cdot 2 - 6}{2^3} = \frac{8-4-6}{8} = -\frac{2}{8} = -\frac{1}{4}$$

$C'(n)$ is the (instantaneous) change in cost. Thus $C'(2) = -\frac{1}{4}$ means the (instantaneous) change in cost of producing 2000 copies of a book is decreasing at a rate of \$.25 per additional copy. $\boxed{}$

149. According to Newton's law of universal gravitation, the force F between two bodies of constant mass m_1 and m_2 is $F = \frac{Gm_1m_2}{r^2}$ where G is the gravitation constant and r is the distance between the bodies.

(A) Find the rate of change of force with respect to distance.

(B) Find the rate of change of gravitational force acting on two bodies of mass 1000 kg when they are 10 meters apart (assume $G = 6.67 \cdot 10^{-11}\ Nm^2/kg^2$).

(A) $\frac{dF}{dr} = \frac{d}{dr}\left[\frac{Gm_1m_2}{r^2}\right] = \frac{d}{dr}\left[Gm_1m_2r^{-2}\right] = -2Gm_1m_2r^{-3}$

(B) $F'(10) = -2 \cdot 6.67 \cdot 10^{-11}\ Nm^2/kg^2 \cdot 10^3 kg \cdot 10^3 kg \cdot (10m)^{-3}$

$= -2 \cdot 6.67 \cdot 10^{-11+3+3-3} \left(\frac{Nm^2 kg^2}{kg^2 m^3}\right)$

$\begin{array}{r} 6.67 \\ \times\ 2 \\ \hline 12.34 \end{array}$

$= -12.34 \cdot 10^{-8}\ N/m = -1.234 \cdot 10^{-7}\ N/m$

$\therefore$ the gravitational force is decreasing at a rate of $1.234 \cdot 10^{-7}\ N/m$ $\boxed{}$