Slopes of Secant Lines

1-10: Calculate the slope of the secant line of $f(x)$ from x_1 to x_2.

Recall: $m = \frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

1. $f(x) = 4x + 7$, $x_1 = 2$, $x_2 = 5$

Sol: $m = \frac{(4 \cdot 5 + 7) - (4 \cdot 2 + 7)}{5 - 2} = \frac{20 + 7 - 8 - 7}{3} = \frac{12}{3} = 4$ ☑

2. $f(x) = 8x - 3$, $x_1 = -1$, $x_2 = 3$

Sol: $m = \frac{(8 \cdot 3 - 3) - (8 \cdot (-1) - 3)}{3 - (-1)} = \frac{24 - 3 - (-8) + 3}{4} = \frac{32}{4} = 8$ ☑

3. $f(x) = x^2 + 2x + 1$, $x_1 = 3$, $x_2 = 3.5$ (Note: $3.5 = \frac{7}{2}$)

Sol: $m = \frac{\left(\left(\frac{7}{2}\right)^2 + 2\left(\frac{7}{2}\right) + 1\right) - (3^2 + 2 \cdot 3 + 1)}{\frac{7}{2} - 3} = \frac{\frac{49}{4} + \frac{28}{4} + 1 - 9 - 6 - 1}{\frac{1}{2}}$

$= \frac{\frac{77}{4} - 15}{\frac{1}{2}} = \frac{77 - 60}{4} \cdot \frac{2}{1} = \frac{17}{2}$ ☑

4. $f(x) = -x^2 + x + 2$, $x_1 = .5$, $x_2 = 1.5$ (NB: $1.5 = \frac{3}{2}$, $.5 = \frac{1}{2}$)

Sol: $m = \frac{\left(-\left(\frac{3}{2}\right)^2 + \frac{3}{2} + 2\right) - \left(-\left(\frac{1}{2}\right)^2 + \frac{1}{2} + 2\right)}{\frac{3}{2} - \frac{1}{2}} = \frac{-\frac{9}{4} + \frac{3}{2} + 2 + \frac{1}{4} - \frac{1}{2} - 2}{1}$

$= -\frac{8}{4} + \frac{2}{2} = -1$ ☑

5. $f(x) = \frac{4}{3x - 1}$, $x_1 = 1$, $x_2 = 3$.

Sol: $m = \frac{\frac{4}{3 \cdot 3 - 1} - \frac{4}{3 \cdot 1 - 1}}{3 - 1} = \frac{\frac{4}{8} - \frac{4}{2}}{2} = \left(\frac{1}{2} - \frac{4}{2}\right) \cdot \frac{1}{2} = \frac{-3}{4}$ ☑

6. $f(x) = \frac{x - 7}{2x + 1}$, $x_1 = 0$, $x_2 = 2$.

Sol: $m = \frac{\frac{2 - 7}{2 \cdot 2 + 1} - \frac{0 - 7}{2 \cdot 0 + 1}}{2 - 0} = \frac{\frac{-5}{5} - \frac{-7}{1}}{2} = \frac{-1 + 7}{2} = 3$ ☑

7. $f(x) = \sqrt{x}$, $x_1 = 1$, $x_2 = 16$

Sol: $m = \dfrac{\sqrt{16} - \sqrt{1}}{16-1} = \dfrac{4-1}{16-1} = \dfrac{3}{15} = \dfrac{1}{5}$

8. $f(x) = \sqrt{x-9}$, $x_1 = 10$, $x_2 = 13$.

Sol: $m = \dfrac{\sqrt{10-9} - \sqrt{13-9}}{10-13} = \dfrac{\sqrt{1} - \sqrt{4}}{-3} = \dfrac{1-2}{-3} = \dfrac{1}{3}$

9. $f(x) = x^{1/3} + 1$, $x_1 = 0$, $x_2 = 8$.

Sol: $m = \dfrac{(8^{1/3}+1) - (0^{1/3}+1)}{8-0} = \dfrac{2+1-1}{8} = \dfrac{1}{4}$

10. $f(x) = 6x^{2/3} + 2x^{1/3}$, $x_1 = 1$, $x_2 = 27$.

Sol: $m = \dfrac{(6 \cdot 27^{2/3} + 2 \cdot 27^{1/3}) - (6 \cdot 1^{2/3} + 2 \cdot 1^{1/3})}{27-1}$

$= \dfrac{(6 \cdot (3^3)^{2/3} + 2 \cdot (3^3)^{1/3}) - (6+2)}{26}$

$= \dfrac{6 \cdot 3^2 + 2 \cdot 3^1 - 8}{26} = \dfrac{6 \cdot 9 + 6 - 8}{26}$

$= \dfrac{54-2}{26} = \dfrac{52}{26} = 2$

Tangent Lines

11–20: Calculate the slope of the tangent line to $f(x)$ at a. Use this to calculate an equation of the tangent line.

Recall: Lines often have many equations...

① $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ is the slope of the tangent line to f at a.

② Lines have point-slope form $y - y_1 = m(x - x_1)$

③ Putting these two together gives us a formula for the tangent line: $y - f(a) = f'(a)(x-a)$.

11. $f(x) = 3-4x$, $a = 2$

Sol: $f(2) = 3 - 4\cdot 2 = 3 - 8 = -5$

$$f'(2) = \lim_{x\to 2}\frac{f(x)-f(2)}{x-2} = \lim_{x\to 2}\frac{(3-4x)-(-5)}{x-2}$$

$$= \lim_{x\to 2}\frac{8-4x}{x-2} = \lim_{x\to 2}\frac{-4(x-2)}{x-2} = \lim_{x\to 2} -4 = -4$$

$\therefore$ Tangent line has equation $y-(-5) = -4(x-2)$ ☒

12. $f(x) = \frac{x}{5}+6$, $a = -1$

Sol: $f(-1) = \frac{-1}{5} + 6 = \frac{29}{5}$

$$f'(-1) = \lim_{x\to -1}\frac{f(x)-f(-1)}{x-(-1)} = \lim_{x\to -1}\frac{\left(\frac{x}{5}+6\right)-\left(\frac{-1}{5}+6\right)}{x+1}$$

$$= \lim_{x\to -1}\frac{\frac{x+1}{5}}{x+1} = \lim_{x\to -1}\frac{x+1}{5(x+1)} = \lim_{x\to -1}\frac{1}{5} = \frac{1}{5}$$

$\therefore$ tangent line is $y - \frac{21}{5} = \frac{1}{5}(x-(-1))$ ☒

13. $f(x) = x^2+x$, $a=1$

Sol: $f(1) = 1^2+1 = 2$

$$f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{((1+h)^2+(1+h))-2}{h} = \lim_{h\to 0}\frac{1+2h+h^2+1+h-2}{h}$$

$$= \lim_{h\to 0}\frac{h(3+h)}{h} = \lim_{h\to 0}(3+h) = 3+0 = 3$$

$\therefore$ tangent line is $y-2 = 3(x-1)$ ☒

14. $f(x) = 1 - x - x^2$, $a = 0$

Sol: $f(0) = 1 - 0 - 0^2 = 1$

$$f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{1-x-x^2-1}{x} = \lim_{x\to 0} \frac{x(-1-x)}{x} = \lim_{x\to 0}(-1-x) = -1-0 = -1$$

$\therefore$ tangent line is $y - 1 = -1(x-0)$ $\boxed{\ }$

15. $f(x) = \frac{7}{x}$, $a = 3$

Sol: $$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{\frac{7}{3+h} - \frac{7}{3}}{h} \cdot \frac{3(3+h)}{3(3+h)} = \lim_{h\to 0} \frac{7\cdot 3 - 7(3+h)}{3h(3+h)}$$

$$= \lim_{h\to 0} \frac{7(3-3-h)}{3h(3+h)} = \lim_{h\to 0} \frac{7(-h)}{3h(3+h)} = \lim_{h\to 0} \frac{-7}{3(3+h)} = -\frac{7}{3(3+0)} = -\frac{7}{9}.$$

$\therefore$ tangent line is $y - \frac{7}{3} = -\frac{7}{9}(x-3)$. $\boxed{\ }$

16. $f(x) = \sqrt{x+8}$, $a = 1$

Sol: $f(1) = \sqrt{1+8} = \sqrt{9} = 3$

$$f'(1) = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{\sqrt{x+8}-3}{x-1} \cdot \frac{\sqrt{x+8}+3}{\sqrt{x+8}+3} = \lim_{x\to 1} \frac{(x+8)-9}{(x-1)(\sqrt{x+8}+3)}$$

$$= \lim_{x\to 1} \frac{x-1}{(x-1)(\sqrt{x+8}+3)} = \lim_{x\to 1} \frac{1}{\sqrt{x+8}+3} = \frac{1}{\sqrt{1+8}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

$\therefore$ tangent line is $y - 3 = \frac{1}{6}(x-1)$ $\boxed{\ }$

17. $f(x) = 2 - 3x^2$, $a = -2$

Sol: $f(-2) = 2 - 3(-2)^2 = 2 - 12 = -10$

$$f'(-2) = \lim_{h\to 0} \frac{f(-2+h)-f(-2)}{h} = \lim_{h\to 0} \frac{2-3(-2+h)^2-(-10)}{h}$$

$$= \lim_{h\to 0} \frac{2-3(4-4h+h^2)+10}{h} = \lim_{h\to 0} \frac{2-12+12h-3h^2+10}{h}$$

$$= \lim_{h\to 0} \frac{h(12-3h)}{h} = \lim_{h\to 0}(12-3h) = 12 - 3\cdot 0 = 12$$

$\therefore$ tangent line is $y-(-10) = 12(x-(-2))$ $\boxed{\ }$

18. $f(x) = -\frac{3}{x-1}$, $a = 4$

Sol: $f(4) = -\frac{3}{4-1} = -1$

$$f'(4) = \lim_{x\to 4} \frac{-\frac{3}{x-1} - (-1)}{x-4} \cdot \frac{x-1}{x-1} = \lim_{x\to 4} \frac{-3+(x-1)}{(x-4)(x-1)} = \lim_{x\to 4} \frac{x-4}{(x-4)(x-1)}$$

$$= \lim_{x\to 4} \frac{1}{x-1} = \frac{1}{4-1} = \frac{1}{3}$$

$\therefore$ tangent line is $y-(-1) = \frac{1}{3}(x-4)$ $\boxed{\ }$

19. $f(x) = \dfrac{2}{x+3}$, $a = -4$

Sol: $f(-4) = \dfrac{2}{-4+3} = -2$

$$f'(-4) = \lim_{h\to 0} \frac{\frac{2}{-4+h+3} - (-2)}{h} = \lim_{h\to 0} \frac{\frac{2}{h-1} + 2}{h} \cdot \frac{h-1}{h-1} = \lim_{h\to 0} \frac{2 + 2(h-1)}{h(h-1)}$$

$$= \lim_{h\to 0} \frac{2(1+h-1)}{h(h-1)} = \lim_{h\to 0} \frac{2h}{h(h-1)} = \lim_{h\to 0} \frac{2}{h-1} = \frac{2}{0-1} = -2$$

$\therefore$ tangent line is $y - (-2) = -2(x-(-4))$ $\square$

20. $f(x) = \dfrac{3}{x^2}$, $a = 3$

Sol: $f(3) = \dfrac{3}{3^2} = \dfrac{1}{3}$

$$f'(3) = \lim_{x\to 3} \frac{f(x) - f(3)}{x-3} = \lim_{x\to 3} \frac{\frac{3}{x^2} - \frac{1}{3}}{x-3} \cdot \frac{3x^2}{3x^2} = \lim_{x\to 3} \frac{3\cdot 3 - x^2}{3x^2(x-3)}$$

$$= \lim_{x\to 3} \frac{(3-x)(3+x)}{3x^2(x-3)} = \lim_{x\to 3} -\frac{(x-3)(x+3)}{3x^2(x-3)} = \lim_{x\to 3} \frac{x+3}{3x^2} = \frac{3+3}{3\cdot 3^2} = \frac{2}{9}$$

$\therefore$ tangent line is $y - \frac{1}{3} = \frac{2}{9}(x-3)$ $\square$

LIMIT DEFINITION OF THE DERIVATIVE

21–30: Calculate $f'(a)$ using the limit definition.

Recall: $f'(a)$ has two formulas:

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} \quad \text{and} \quad f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

21. $f(x) = 5x+4$, $a=-1$

Sol: $f'(-1) = \lim_{x\to -1} \frac{(5x+4)-(5(-1)+4)}{x-(-1)} = \lim_{x\to -1} \frac{5(x-(-1))}{x-(-1)} = \lim_{x\to -1} 5 = 5.$ $\boxed{\checkmark}$

22. $f(x) = -7x+1$, $a=3$

Sol: $f'(3) = \lim_{h\to 0} \frac{(-7(3+h)+1)-(-7\cdot 3+1)}{h} = \lim_{h\to 0} \frac{-7(3+h-3)}{h} = \lim_{h\to 0} \frac{-7h}{h} = \lim_{h\to 0} -7 = -7$ $\boxed{\checkmark}$

23. $f(x) = x^2+9x$, $a=2$

Sol: $f'(2) = \lim_{x\to 2} \frac{(x^2+9x)-(2^2+9\cdot 2)}{x-2} = \lim_{x\to 2} \frac{(x^2-2^2)+(9x-9\cdot 2)}{x-2}$

$= \lim_{x\to 2} \frac{(x-2)(x+2)+(x-2)\cdot 9}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2+9)}{x-2}$

$= \lim_{x\to 2} (x+11) = 2+11 = 13$ $\boxed{\checkmark}$

24. $f(x) = 3x^2-x+2$, $a=1$

Sol: $f'(1) = \lim_{h\to 0} \frac{(3(1+h)^2-(1+h)+2)-(3\cdot 1^2-1+2)}{h}$

$= \lim_{h\to 0} \frac{3((1+h)^2-1^2)-((1+h)-1)}{h}$

$= \lim_{h\to 0} \frac{3(1+h+1)(1+h-1)-h}{h} = \lim_{h\to 0} \frac{3h(h+2)-h}{h}$

$= \lim_{h\to 0} \frac{h(3(h+2)-1)}{h} = \lim_{h\to 0} (3(h+2)-1) = 3(0+2)-1 = 5$ $\boxed{\checkmark}$

25. $f(x) = \sqrt{x}$, $a=4$

Sol: $f'(4) = \lim_{h\to 0} \frac{\sqrt{4+h}-\sqrt{4}}{h} = \lim_{h\to 0} \frac{\sqrt{4+h}-2}{h} \cdot \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$

$= \lim_{h\to 0} \frac{(4+h)-4}{h(\sqrt{4+h}+2)} = \lim_{h\to 0} \frac{h}{h(\sqrt{4+h}+2)} = \lim_{h\to 0} \frac{1}{\sqrt{4+h}+2} = \frac{1}{\sqrt{4+0}+2} = \frac{1}{4}$ $\boxed{\checkmark}$

26: $f(x) = \sqrt{x-2}$, $a=6$

Sol: $f'(6) = \lim_{x\to 6} \frac{\sqrt{x-2}-\sqrt{6-2}}{x-6} = \lim_{x\to 6} \frac{\sqrt{x-2}-2}{x-6} \cdot \frac{\sqrt{x-2}+2}{\sqrt{x-2}+2}$

$= \lim_{x\to 6} \frac{(x-2)-4}{(x-6)(\sqrt{x-2}+2)} = \lim_{x\to 6} \frac{x-6}{(x-6)(\sqrt{x-2}+2)} = \lim_{x\to 6} \frac{1}{\sqrt{x-2}+2} = \frac{1}{\sqrt{6-2}+2} = \frac{1}{4}$ $\boxed{\checkmark}$

27. $f(x)=\frac{1}{x}$, $a=2$

Sol: $f'(2)=\lim_{x\to 2}\frac{\frac{1}{x}-\frac{1}{2}}{x-2}\cdot\frac{2x}{2x}=\lim_{x\to 2}\frac{2-x}{2x(x-2)}=\lim_{x\to 2}\frac{-(x-2)}{2x(x-2)}$

$=\lim_{x\to 2}\frac{-1}{2x}=\frac{-1}{2\cdot 2}=-\frac{1}{4}$ $\boxed{\text{Q.E.D.}}$

28. $f(x)=\frac{1}{x-3}$, $a=-1$

Sol: $f'(-1)=\lim_{x\to -1}\frac{\frac{1}{x-3}-\frac{1}{-1-3}}{x-(-1)}=\lim_{x\to -1}\frac{\frac{1}{x-3}+\frac{1}{4}}{x+1}\cdot\frac{4(x-3)}{4(x-3)}$

$=\lim_{x\to -1}\frac{4+(x-3)}{4(x+1)(x-3)}=\lim_{x\to -1}\frac{x+1}{4(x+1)(x-3)}$

$=\lim_{x\to -1}\frac{1}{4(x-3)}=\frac{1}{4(-1-3)}=-\frac{1}{16}$ $\boxed{\text{Q.E.D.}}$

29. $f(x)=\frac{1}{x^3}$, $a=1$

Sol: $f'(1)=\lim_{h\to 0}\frac{\frac{1}{(1+h)^3}-\frac{1}{1^3}}{h}\cdot\frac{(1+h)^3}{(1+h)^3}$

$=\lim_{h\to 0}\frac{1-(1+h)^3}{h(1+h)^3}=\lim_{h\to 0}\frac{1-(1+3h+3h^2+h^3)}{h(1+h)^3}$

$=\lim_{h\to 0}\frac{-h(3+3h+h^2)}{h(1+h)^3}=\lim_{h\to 0}-\frac{(3+3h+h^2)}{(1+h)^3}$

$=-\frac{(3+3\cdot 0+0^2)}{(1+0)^3}=-\frac{3}{1}=-3$ $\boxed{\text{Q.E.D.}}$

30. $f(x)=\frac{1}{\sqrt{x}}$, $a=4$

Sol: $f'(4)=\lim_{x\to 4}\frac{\frac{1}{\sqrt{x}}-\frac{1}{\sqrt{4}}}{x-4}=\lim_{x\to 4}\frac{\frac{1}{\sqrt{x}}-\frac{1}{2}}{x-4}\cdot\frac{2\sqrt{x}}{2\sqrt{x}}$

$=\lim_{x\to 4}\frac{2-\sqrt{x}}{2\sqrt{x}(x-4)}\cdot\frac{2+\sqrt{x}}{2+\sqrt{x}}=\lim_{x\to 4}\frac{4-x}{2\sqrt{x}(x-4)(2+\sqrt{x})}$

$=\lim_{x\to 4}\frac{-(x-4)}{2\sqrt{x}(x-4)(2+\sqrt{x})}=\lim_{x\to 4}\frac{-1}{2\sqrt{x}(2+\sqrt{x})}$

$=\frac{-1}{2\sqrt{4}(2+\sqrt{4})}=\frac{-1}{2\cdot 2\cdot(2+2)}=-\frac{1}{16}$ $\boxed{\text{Q.E.D.}}$

41–44. Show that $f'(a)$ does not exist at the given a.

41. $f(x) = x^{1/3}$, $a = 0$

Sol: $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{(a+h)^{1/3}-a^{1/3}}{h} \cdot \frac{(a+h)^{2/3}+(a+h)^{1/3}a^{1/3}+a^{2/3}}{(a+h)^{2/3}+(a+h)^{1/3}a^{1/3}+a^{2/3}}$

IDEA: $x^3-y^3 = (x-y)(x^2+xy+y^2)$
So $x=(a+h)^{1/3}$, $y = a^{1/3}$

$$= \lim_{h\to 0} \frac{(a+h)-a}{h\left((a+h)^{2/3}+(a+h)^{1/3}a^{1/3}+a^{2/3}\right)}$$

$$= \lim_{h\to 0} \frac{h}{h\left((a+h)^{2/3}+(a+h)^{1/3}a^{1/3}+a^{2/3}\right)}$$

$$= \lim_{h\to 0} \frac{1}{(a+h)^{2/3}+(a+h)^{1/3}a^{1/3}+a^{2/3}} = \frac{1}{(a+0)^{2/3}+(a+0)^{1/3}a^{1/3}+a^{2/3}}$$

$$= \frac{1}{3a^{2/3}}$$ BUT: if $a=0$ this is undefined! ☒

42. $f(x) = x^{2/3}$, $a = 0$

Sol: $f'(a) = \lim_{x\to a} \frac{x^{2/3}-a^{2/3}}{x-a} \cdot \frac{(x^{2/3})^2 + x^{2/3}a^{2/3} + (a^{2/3})^2}{(x^{2/3})^2 + x^{2/3}a^{2/3} + (a^{2/3})^2}$ ← IDEA: Same as above

$$= \lim_{x\to a} \frac{x-a}{(x-a)(x^{4/3}+x^{2/3}a^{2/3}+a^{4/3})}$$

$$= \lim_{x\to a} \frac{1}{x^{4/3}+x^{2/3}a^{2/3}+a^{4/3}} = \frac{1}{a^{4/3}+a^{2/3}\cdot a^{2/3}+a^{4/3}} = \frac{1}{3a^{4/3}}$$

BUT this expression is undefined if $a=0$... ☒

43. $f(x) = \begin{cases} 1 & \text{if } x<1 \\ x & \text{if } x \ge 1 \end{cases}$, $a = 1$

Sol: $f'(1) = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1}$. We show $\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} \neq \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1}$.

Left: $\lim_{x\to 1^-} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^-} \frac{1-1}{x-1} = \lim_{x\to 1^-} 0 = 0$

Right: $\lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^+} \frac{x-1}{x-1} = \lim_{x\to 1^+} 1 = 1$

But $0 \neq 1$, so $f'(1)$ does not exist! ☒

44. $f(x) = \frac{|x|}{x}$, $a = 0$

Sol: $f(0)$ is undefined, so $f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0}$ is a meaningless expression! ☒

Physicsy Questions

35–38: A particle moves along a straight line. The position of the particle at time t is given by the function $S(t)$. First write an expression that describes the average velocity of the particle from time $t=2$ to time $t=2+h$. Then calculate the instantaneous velocity of the particle at two seconds.

35. $S(t) = \frac{1}{3}t + 5$

Average: $\frac{S(2+h)-S(2)}{(2+h)-2} = \frac{\left(\frac{1}{3}(2+h)+5\right)-\left(\frac{1}{3}2+5\right)}{h} = \frac{\frac{1}{3}(2+h-2)}{h} = \frac{\frac{1}{3}h}{h} = \frac{1}{3}$

Instantaneous: $V(2) = S'(2) = \lim_{h\to 0} \frac{S(2+h)-S(2)}{h} = \lim_{h\to 0} \frac{1}{3} = \frac{1}{3}$ ☒

36. $S(t) = t^2 - 2t$

Average: $\frac{S(2+h)-S(2)}{(2+h)-2} = \frac{\left((2+h)^2 - 2(2+h)\right) - \left(2^2 - 2\cdot 2\right)}{h}$

$= \frac{(4+4h+h^2-4-2h)-0}{h}$

$= \frac{h(4+h-2)}{h} = 2+h$

Instantaneous: $V(2) = S'(2) = \lim_{h\to 0} \frac{S(2+h)-S(2)}{(2+h)-2} = \lim_{h\to 0}(2+h) = 2+0 = 2$ ☒

37. $S(t) = 2t^3 + 3$

Average: $\frac{S(2+h)-S(2)}{(2+h)-2} = \frac{\left(2(2+h)^3+3\right) - \left(2\cdot 2^3+3\right)}{h}$

$= \frac{2(8+6h+12h^2+h^3)+3-16-3}{h}$

$= \frac{16+12h+24h^2+2h^3-16}{h}$

$= \frac{h(12+24h+2h^2)}{h} = 12+24h+2h^2$

Instantaneous: $V(2) = S'(2) = \lim_{h\to 0} \frac{S(2+h)-S(2)}{(2+h)-2} = \lim_{h\to 0}(12+24h+2h^2) = 12+24\cdot 0+2\cdot 0^2 = 12$ ☒

38. $S(t) = \frac{16}{t^2} - \frac{4}{t}$

Average: $\frac{S(2+h)-S(2)}{(2+h)-2} = \frac{\left(\frac{16}{(2+h)^2} - \frac{4}{2+h}\right) - \left(\frac{16}{2^2} - \frac{4}{2}\right)}{h} \cdot \frac{(2+h)^2}{(2+h)^2} = \frac{16 - 4(2+h) - (4-2)(2+h)^2}{h(2+h)^2}$

$= \frac{16-8-4h-2(4+4h+h^2)}{h(2+h)^2} = \frac{8-4h-8-8h-2h^2}{h(2+h)^2}$

$= \frac{-h(12+2h)}{h(2+h)^2} = \frac{-(12+2h)}{(2+h)^2}$

Instantaneous: $V(2) = S'(2) = \lim_{h\to 0} \frac{S(2+h)-S(2)}{(2+h)-2} = \lim_{h\to 0} -\frac{12+2h}{(2+h)^2} = -\frac{12+2\cdot 0}{(2+0)^2} = -3$ ☒