

# SLOPES OF SECANT LINES

1-10: Calculate the slope of the secant line of  $f(x)$  from  $x_1$  to  $x_2$ .

Recall:  $m = \frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

1.  $f(x) = 4x + 7$ ,  $x_1 = 2$ ,  $x_2 = 5$

Sol:  $m = \frac{(4 \cdot 5 + 7) - (4 \cdot 2 + 7)}{5 - 2} = \frac{20 + 7 - 8 - 7}{3} = \frac{12}{3} = 4 \quad \square$

2.  $f(x) = 8x - 3$ ,  $x_1 = -1$ ,  $x_2 = 3$

Sol:  $m = \frac{(8 \cdot 3 - 3) - (8 \cdot (-1) - 3)}{3 - (-1)} = \frac{24 - 3 - (-8) - 3}{4} = \frac{32}{4} = 8 \quad \square$

3.  $f(x) = x^2 + 2x + 1$ ,  $x_1 = 3$ ,  $x_2 = 3.5$  (Note:  $3.5 = \frac{7}{2}$ )

Sol:  $m = \frac{((\frac{7}{2})^2 + 2(\frac{7}{2}) + 1) - (3^2 + 2 \cdot 3 + 1)}{\frac{7}{2} - 3} = \frac{\frac{49}{4} + \frac{28}{4} + 1 - 9 - 6 - 1}{\frac{1}{2}}$   
 $= \frac{\frac{77}{4} - 15}{\frac{1}{2}} = \frac{77 - 60}{4} \cdot \frac{2}{1} = \frac{17}{2} \quad \square$

4.  $f(x) = -x^2 + x + 2$ ,  $x_1 = .5$ ,  $x_2 = 1.5$  (NB:  $1.5 = \frac{3}{2}$ ,  $.5 = \frac{1}{2}$ )

Sol:  $m = \frac{(-(\frac{3}{2})^2 + \frac{3}{2} + 2) - (-\frac{1}{2}^2 + \frac{1}{2} + 2)}{\frac{3}{2} - \frac{1}{2}} = \frac{-\frac{9}{4} + \frac{3}{2} + 2 + \frac{1}{4} - \frac{1}{2} - 2}{1}$   
 $= -\frac{8}{4} + \frac{2}{2} = -1 \quad \square$

5.  $f(x) = \frac{4}{3x-1}$ ,  $x_1 = 1$ ,  $x_2 = 3$ .

Sol:  $m = \frac{\frac{4}{3 \cdot 3 - 1} - \frac{4}{3 \cdot 1 - 1}}{3 - 1} = \frac{\frac{4}{8} - \frac{4}{2}}{2} = (\frac{1}{2} - \frac{4}{2}) \cdot \frac{1}{2} = -\frac{3}{4} \quad \square$

6.  $f(x) = \frac{x-7}{2x+1}$ ,  $x_1 = 0$ ,  $x_2 = 2$ .

Sol:  $m = \frac{\frac{2-7}{2 \cdot 2 + 1} - \frac{0-7}{2 \cdot 0 + 1}}{2 - 0} = \frac{\frac{-5}{5} - \frac{-7}{1}}{2} = \frac{-1 + 7}{2} = 3 \quad \square$

7.  $f(x) = \sqrt{x}$ ,  $x_1 = 1$ ,  $x_2 = 16$

Sol:  $m = \frac{\sqrt{16} - \sqrt{1}}{16 - 1} = \frac{4 - 1}{16 - 1} = \frac{3}{15} = \frac{1}{5}$   $\square$

8.  $f(x) = \sqrt{x-9}$ ,  $x_1 = 10$ ,  $x_2 = 13$ .

Sol:  $m = \frac{\sqrt{10-9} - \sqrt{13-9}}{10 - 13} = \frac{\sqrt{1} - \sqrt{4}}{-3} = \frac{1 - 2}{-3} = \frac{1}{3}$   $\square$

9.  $f(x) = x^{1/3} + 1$ ,  $x_1 = 0$ ,  $x_2 = 8$ .

Sol:  $m = \frac{(8^{1/3} + 1) - (0^{1/3} + 1)}{8 - 0} = \frac{2 + 1 - 1}{8} = \frac{1}{4}$   $\square$

10.  $f(x) = 6x^{2/3} + 2x^{1/3}$ ,  $x_1 = 1$ ,  $x_2 = 27$ .

Sol:  $m = \frac{(6 \cdot 27^{2/3} + 2 \cdot 27^{1/3}) - (6 \cdot 1^{2/3} + 2 \cdot 1^{1/3})}{27 - 1}$   
 $= \frac{(6 \cdot (3^3)^{2/3} + 2 \cdot (3^3)^{1/3}) - (6 + 2)}{26}$   
 $= \frac{6 \cdot 3^2 + 2 \cdot 3 - 8}{26} = \frac{6 \cdot 9 + 6 - 8}{26}$   
 $= \frac{54 - 2}{26} = \frac{52}{26} = 2$   $\square$

# TANGENT LINES

11-20: Calculate the slope of the tangent line to  $f(x)$  at  $a$ .  
Use this to calculate an equation of the tangent line.

Recall: Lines often have many equations...

①  $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$  is the slope of the tangent line to  $f$  at  $a$ .

② Lines have point-slope form  $y - y_1 = m(x - x_1)$

③ Putting these two together gives us a formula for the tangent line:  $y - f(a) = f'(a)(x - a)$ .

11.  $f(x) = 3 - 4x$ ,  $a = 2$

Sol:  $f(2) = 3 - 4 \cdot 2 = 3 - 8 = -5$

$$f'(2) = \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2} \frac{(3 - 4x) - (-5)}{x - 2}$$
$$= \lim_{x \rightarrow 2} \frac{8 - 4x}{x - 2} = \lim_{x \rightarrow 2} \frac{-4(x - 2)}{x - 2} = \lim_{x \rightarrow 2} -4 = -4$$

$\therefore$  Tangent line has equation  $y - (-5) = -4(x - 2)$   $\square$

12.  $f(x) = \frac{x}{5} + 6$ ,  $a = -1$

Sol:  $f(-1) = \frac{-1}{5} + 6 = \frac{29}{5}$

$$f'(-1) = \lim_{x \rightarrow -1} \frac{f(x) - f(-1)}{x - (-1)} = \lim_{x \rightarrow -1} \frac{(\frac{x}{5} + 6) - (\frac{-1}{5} + 6)}{x + 1}$$
$$= \lim_{x \rightarrow -1} \frac{\frac{x+1}{5}}{x+1} = \lim_{x \rightarrow -1} \frac{x+1}{5(x+1)} = \lim_{x \rightarrow -1} \frac{1}{5} = \frac{1}{5}$$

$\therefore$  tangent line is  $y - \frac{29}{5} = \frac{1}{5}(x - (-1))$   $\square$

13.  $f(x) = x^2 + x$ ,  $a = 1$

Sol:  $f(1) = 1^2 + 1 = 2$

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{((1+h)^2 + (1+h)) - 2}{h} = \lim_{h \rightarrow 0} \frac{1 + 2h + h^2 + 1 + h - 2}{h}$$
$$= \lim_{h \rightarrow 0} \frac{h(3+h)}{h} = \lim_{h \rightarrow 0} (3+h) = 3 + 0 = 3$$

$\therefore$  tangent line is  $y - 2 = 3(x - 1)$   $\square$

14.  $f(x) = 1 - x - x^2$ ,  $a = 0$

Sol:  $f(0) = 1 - 0 - 0^2 = 1$

$$f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0} \frac{1 - x - x^2 - 1}{x} = \lim_{x \rightarrow 0} \frac{x(-1 - x)}{x} = \lim_{x \rightarrow 0} (-1 - x) = -1 - 0 = -1$$

$\therefore$  tangent line is  $y - 1 = -1(x - 0)$   $\square$

15.  $f(x) = \frac{7}{x}$ ,  $a = 3$

$$\begin{aligned} \text{Sol: } f'(3) &= \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \rightarrow 0} \frac{\frac{7}{3+h} - \frac{7}{3}}{h} \cdot \frac{3(3+h)}{3(3+h)} = \lim_{h \rightarrow 0} \frac{7 \cdot 3 - 7(3+h)}{3h(3+h)} \\ &= \lim_{h \rightarrow 0} \frac{7(3 - 3 - h)}{3h(3+h)} = \lim_{h \rightarrow 0} \frac{7(-h)}{3h(3+h)} = \lim_{h \rightarrow 0} \frac{-7}{3(3+h)} = \frac{-7}{3(3+0)} = -\frac{7}{9} \end{aligned}$$

$\therefore$  tangent line is  $y - \frac{7}{3} = -\frac{7}{9}(x - 3)$   $\square$

16.  $f(x) = \sqrt{x+8}$ ,  $a = 1$

Sol:  $f(1) = \sqrt{1+8} = \sqrt{9} = 3$

$$\begin{aligned} f'(1) &= \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1} \frac{\sqrt{x+8} - 3}{x - 1} \cdot \frac{\sqrt{x+8} + 3}{\sqrt{x+8} + 3} = \lim_{x \rightarrow 1} \frac{(x+8) - 9}{(x-1)(\sqrt{x+8} + 3)} \\ &= \lim_{x \rightarrow 1} \frac{x-1}{(x-1)(\sqrt{x+8} + 3)} = \lim_{x \rightarrow 1} \frac{1}{\sqrt{x+8} + 3} = \frac{1}{\sqrt{1+8} + 3} = \frac{1}{\sqrt{9} + 3} = \frac{1}{6} \end{aligned}$$

$\therefore$  tangent line is  $y - 3 = \frac{1}{6}(x - 1)$   $\square$

17.  $f(x) = 2 - 3x^2$ ,  $a = -2$

Sol:  $f(-2) = 2 - 3(-2)^2 = 2 - 12 = -10$

$$\begin{aligned} f'(-2) &= \lim_{h \rightarrow 0} \frac{f(-2+h) - f(-2)}{h} = \lim_{h \rightarrow 0} \frac{2 - 3(-2+h)^2 - (-10)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2 - 3(4 - 4h + h^2) + 10}{h} = \lim_{h \rightarrow 0} \frac{2 - 12 + 12h - 3h^2 + 10}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(12 - 3h)}{h} = \lim_{h \rightarrow 0} (12 - 3h) = 12 - 3 \cdot 0 = 12 \end{aligned}$$

$\therefore$  tangent line is  $y - (-10) = 12(x - (-2))$   $\square$

18.  $f(x) = -\frac{3}{x-1}$ ,  $a = 4$

Sol:  $f(4) = -\frac{3}{4-1} = -1$

$$\begin{aligned} f'(4) &= \lim_{x \rightarrow 4} \frac{-\frac{3}{x-1} - (-1)}{x-4} \cdot \frac{x-1}{x-1} = \lim_{x \rightarrow 4} \frac{-3 + (x-1)}{(x-4)(x-1)} = \lim_{x \rightarrow 4} \frac{x-4}{(x-4)(x-1)} \\ &= \lim_{x \rightarrow 4} \frac{1}{x-1} = \frac{1}{4-1} = \frac{1}{3} \end{aligned}$$

$\therefore$  tangent line is  $y - (-1) = \frac{1}{3}(x - 4)$   $\square$

$$19. f(x) = \frac{2}{x+3}, \quad a = -4$$

$$\text{Sol: } f(-4) = \frac{2}{-4+3} = -2$$

$$\begin{aligned} f'(-4) &= \lim_{h \rightarrow 0} \frac{\frac{2}{(-4+h)+3} - (-2)}{h} = \lim_{h \rightarrow 0} \frac{\frac{2}{h-1} + 2}{h} \cdot \frac{h-1}{h-1} = \lim_{h \rightarrow 0} \frac{2 + 2(h-1)}{h(h-1)} \\ &= \lim_{h \rightarrow 0} \frac{2(1+h-1)}{h(h-1)} = \lim_{h \rightarrow 0} \frac{2h}{h(h-1)} = \lim_{h \rightarrow 0} \frac{2}{h-1} = \frac{2}{0-1} = -2 \end{aligned}$$

$$\therefore \text{ tangent line is } y - (-2) = -2(x - (-4)) \quad \square$$

$$20. f(x) = \frac{3}{x^2}, \quad a = 3$$

$$\text{Sol: } f(3) = \frac{3}{3^2} = \frac{1}{3}$$

$$\begin{aligned} f'(3) &= \lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3} = \lim_{x \rightarrow 3} \frac{\frac{3}{x^2} - \frac{1}{3}}{x - 3} \cdot \frac{3x^2}{3x^2} = \lim_{x \rightarrow 3} \frac{3 \cdot 3 - x^2}{3x^2(x-3)} \\ &= \lim_{x \rightarrow 3} \frac{(3-x)(3+x)}{3x^2(x-3)} = \lim_{x \rightarrow 3} \frac{-(x-3)(x+3)}{3x^2(x-3)} = \lim_{x \rightarrow 3} \frac{x+3}{3x^2} = \frac{3+3}{3 \cdot 3^2} = \frac{2}{9} \end{aligned}$$

$$\therefore \text{ tangent line is } y - \frac{1}{3} = \frac{2}{9}(x - 3) \quad \square$$

# LIMIT DEFINITION OF THE DERIVATIVE

21-30: Calculate  $f'(a)$  using the limit definition.

Recall:  $f'(a)$  has two formulas:

$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \quad \text{and} \quad f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

21.  $f(x) = 5x + 4$ ,  $a = -1$

Sol:  $f'(-1) = \lim_{x \rightarrow -1} \frac{(5x+4) - (5(-1)+4)}{x - (-1)} = \lim_{x \rightarrow -1} \frac{5(x - (-1))}{x - (-1)} = \lim_{x \rightarrow -1} 5 = 5$   $\square$

22.  $f(x) = -7x + 1$ ,  $a = 3$

Sol:  $f'(3) = \lim_{h \rightarrow 0} \frac{(-7(3+h)+1) - (-7 \cdot 3 + 1)}{h} = \lim_{h \rightarrow 0} \frac{-7(3+h-3)}{h} = \lim_{h \rightarrow 0} \frac{-7h}{h} = \lim_{h \rightarrow 0} -7 = -7$   $\square$

23.  $f(x) = x^2 + 9x$ ,  $a = 2$

Sol:  $f'(2) = \lim_{x \rightarrow 2} \frac{(x^2+9x) - (2^2+9 \cdot 2)}{x-2} = \lim_{x \rightarrow 2} \frac{(x^2-2^2) + (9x-9 \cdot 2)}{x-2}$   
 $= \lim_{x \rightarrow 2} \frac{(x-2)(x+2) + (x-2) \cdot 9}{x-2} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2+9)}{x-2}$   
 $= \lim_{x \rightarrow 2} (x+11) = 2+11 = 13$   $\square$

24.  $f(x) = 3x^2 - x + 2$ ,  $a = 1$

Sol:  $f'(1) = \lim_{h \rightarrow 0} \frac{(3(1+h)^2 - (1+h) + 2) - (3 \cdot 1^2 - 1 + 2)}{h}$   
 $= \lim_{h \rightarrow 0} \frac{3((1+h)^2 - 1^2) - ((1+h) - 1)}{h}$   
 $= \lim_{h \rightarrow 0} \frac{3(1+h+1)(1+h-1) - h}{h} = \lim_{h \rightarrow 0} \frac{3h(h+2) - h}{h}$   
 $= \lim_{h \rightarrow 0} \frac{h(3(h+2) - 1)}{h} = \lim_{h \rightarrow 0} (3(h+2) - 1) = 3(0+2) - 1 = 5$   $\square$

25.  $f(x) = \sqrt{x}$ ,  $a = 4$

Sol:  $f'(4) = \lim_{h \rightarrow 0} \frac{\sqrt{4+h} - \sqrt{4}}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{4+h} - 2}{h} \cdot \frac{\sqrt{4+h} + 2}{\sqrt{4+h} + 2}$   
 $= \lim_{h \rightarrow 0} \frac{(4+h) - 4}{h(\sqrt{4+h} + 2)} = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{4+h} + 2)} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{4+h} + 2} = \frac{1}{\sqrt{4+0} + 2} = \frac{1}{4}$   $\square$

26:  $f(x) = \sqrt{x-2}$ ,  $a = 6$

Sol:  $f'(6) = \lim_{x \rightarrow 6} \frac{\sqrt{x-2} - \sqrt{6-2}}{x-6} = \lim_{x \rightarrow 6} \frac{\sqrt{x-2} - 2}{x-6} \cdot \frac{\sqrt{x-2} + 2}{\sqrt{x-2} + 2}$   
 $= \lim_{x \rightarrow 6} \frac{(x-2) - 4}{(x-6)(\sqrt{x-2} + 2)} = \lim_{x \rightarrow 6} \frac{x-6}{(x-6)(\sqrt{x-2} + 2)} = \lim_{x \rightarrow 6} \frac{1}{\sqrt{x-2} + 2} = \frac{1}{\sqrt{6-2} + 2} = \frac{1}{4}$   $\square$

27.  $f(x) = \frac{1}{x}$ ,  $a = 2$

Sol:  $f'(2) = \lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2} \cdot \frac{2x}{2x} = \lim_{x \rightarrow 2} \frac{2-x}{2x(x-2)} = \lim_{x \rightarrow 2} \frac{-(x-2)}{2x(x-2)}$   
 $= \lim_{x \rightarrow 2} \frac{-1}{2x} = -\frac{1}{2 \cdot 2} = -\frac{1}{4}$   $\square$

28.  $f(x) = \frac{1}{x-3}$ ,  $a = -1$

Sol:  $f'(-1) = \lim_{x \rightarrow -1} \frac{\frac{1}{x-3} - \frac{1}{-1-3}}{x-(-1)} = \lim_{x \rightarrow -1} \frac{\frac{1}{x-3} + \frac{1}{4}}{x+1} \cdot \frac{4(x-3)}{4(x-3)}$   
 $= \lim_{x \rightarrow -1} \frac{4 + (x-3)}{4(x+1)(x-3)} = \lim_{x \rightarrow -1} \frac{x+1}{4(x+1)(x-3)}$   
 $= \lim_{x \rightarrow -1} \frac{1}{4(x-3)} = \frac{1}{4(-1-3)} = -\frac{1}{16}$   $\square$

29.  $f(x) = \frac{1}{x^3}$ ,  $a = 1$

Sol:  $f'(1) = \lim_{h \rightarrow 0} \frac{\frac{1}{(1+h)^3} - \frac{1}{1^3}}{h} \cdot \frac{(1+h)^3}{(1+h)^3}$   
 $= \lim_{h \rightarrow 0} \frac{1 - (1+h)^3}{h(1+h)^3} = \lim_{h \rightarrow 0} \frac{1 - (1 + 3h + 3h^2 + h^3)}{h(1+h)^3}$   
 $= \lim_{h \rightarrow 0} \frac{-h(3 + 3h + h^2)}{h(1+h)^3} = \lim_{h \rightarrow 0} -\frac{(3 + 3h + h^2)}{(1+h)^3}$   
 $= -\frac{(3 + 3 \cdot 0 + 0^2)}{(1+0)^3} = -\frac{3}{1} = -3$   $\square$

30.  $f(x) = \frac{1}{\sqrt{x}}$ ,  $a = 4$

Sol:  $f'(4) = \lim_{x \rightarrow 4} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{4}}}{x-4} = \lim_{x \rightarrow 4} \frac{\frac{1}{\sqrt{x}} - \frac{1}{2}}{x-4} \cdot \frac{2\sqrt{x}}{2\sqrt{x}}$   
 $= \lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{2\sqrt{x}(x-4)} \cdot \frac{2+\sqrt{x}}{2+\sqrt{x}} = \lim_{x \rightarrow 4} \frac{4-x}{2\sqrt{x}(x-4)(2+\sqrt{x})}$   
 $= \lim_{x \rightarrow 4} \frac{-(x-4)}{2\sqrt{x}(x-4)(2+\sqrt{x})} = \lim_{x \rightarrow 4} \frac{-1}{2\sqrt{x}(2+\sqrt{x})}$   
 $= \frac{-1}{2\sqrt{4}(2+\sqrt{4})} = \frac{-1}{2 \cdot 2 \cdot (2+2)} = -\frac{1}{16}$   $\square$

41-44. Show that  $f'(a)$  does not exist at the given  $a$ .

41.  $f(x) = x^{1/3}$ ,  $a=0$

Sol:  $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{(a+h)^{1/3} - a^{1/3}}{h}$

$$= \lim_{h \rightarrow 0} \frac{(a+h)^{2/3} + (a+h)^{1/3} a^{1/3} + a^{2/3}}{h((a+h)^{2/3} + (a+h)^{1/3} a^{1/3} + a^{2/3})}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h((a+h)^{2/3} + (a+h)^{1/3} a^{1/3} + a^{2/3})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{(a+h)^{2/3} + (a+h)^{1/3} a^{1/3} + a^{2/3}} = \frac{1}{(a+0)^{2/3} + (a+0)^{1/3} a^{1/3} + a^{2/3}}$$

$$= \frac{1}{3a^{2/3}}$$

BUT: if  $a=0$  this is undefined!  $\square$

IDEA:  $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$   
 so  $x = (a+h)^{1/3}$ ,  $y = a^{1/3}$

42.  $f(x) = x^{2/3}$ ,  $a=0$

Sol:  $f'(a) = \lim_{x \rightarrow a} \frac{x^{2/3} - a^{2/3}}{x-a} \cdot \frac{(x^{2/3})^2 + x^{2/3} a^{2/3} + (a^{2/3})^2}{(x^{2/3})^2 + x^{2/3} a^{2/3} + (a^{2/3})^2}$

$$= \lim_{x \rightarrow a} \frac{x^{4/3} - a^{4/3}}{(x-a)(x^{4/3} + x^{2/3} a^{2/3} + a^{4/3})}$$

$$= \lim_{x \rightarrow a} \frac{1}{x^{4/3} + x^{2/3} a^{2/3} + a^{4/3}} = \frac{1}{a^{4/3} + a^{2/3} a^{2/3} + a^{4/3}} = \frac{1}{3a^{4/3}}$$

IDEA: Same as above

BUT this expression is undefined if  $a=0$ ...

43.  $f(x) = \begin{cases} 1 & \text{if } x < 1 \\ x & \text{if } x \geq 1 \end{cases}$ ,  $a=1$

Sol:  $f'(1) = \lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x-1}$ . We show  $\lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x-1} \neq \lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x-1}$

Left:  $\lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x-1} = \lim_{x \rightarrow 1^-} \frac{1-1}{x-1} = \lim_{x \rightarrow 1^-} 0 = 0$

Right:  $\lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x-1} = \lim_{x \rightarrow 1^+} \frac{x-1}{x-1} = \lim_{x \rightarrow 1^+} 1 = 1$

But  $0 \neq 1$ , so  $f'(1)$  does not exist!  $\square$

44.  $f(x) = \frac{|x|}{x}$ ,  $a=0$

Sol:  $f(0)$  is undefined, so  $f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x-0}$  is a meaningless expression!  $\square$



# PHYSICS QUESTIONS

35-38: A particle moves along a straight line. The position of the particle at time  $t$  is given by the function  $s(t)$ . First write an expression that describes the average velocity of the particle from time  $t=2$  to time  $t=2+h$ . Then calculate the instantaneous velocity of the particle at two seconds.

35.  $s(t) = \frac{1}{3}t + 5$

Average:  $\frac{s(2+h) - s(2)}{(2+h) - 2} = \frac{(\frac{1}{3}(2+h) + 5) - (\frac{1}{3} \cdot 2 + 5)}{h} = \frac{\frac{1}{3}(2+h-2)}{h} = \frac{\frac{1}{3}h}{h} = \frac{1}{3}$

Instantaneous:  $v(2) = s'(2) = \lim_{h \rightarrow 0} \frac{s(2+h) - s(2)}{h} = \lim_{h \rightarrow 0} \frac{1}{3} = \frac{1}{3}$   $\square$

36.  $s(t) = t^2 - 2t$

Average:  $\frac{s(2+h) - s(2)}{(2+h) - 2} = \frac{((2+h)^2 - 2(2+h)) - (2^2 - 2 \cdot 2)}{h}$   
 $= \frac{(4 + 4h + h^2 - 4 - 2h) - 0}{h}$   
 $= \frac{h(4 + h - 2)}{h} = 2 + h$

Instantaneous:  $v(2) = s'(2) = \lim_{h \rightarrow 0} \frac{s(2+h) - s(2)}{(2+h) - 2} = \lim_{h \rightarrow 0} (2+h) = 2+0 = 2$   $\square$

37.  $s(t) = 2t^3 + 3$

Average:  $\frac{s(2+h) - s(2)}{(2+h) - 2} = \frac{(2(2+h)^3 + 3) - (2 \cdot 2^3 + 3)}{h}$   
 $= \frac{2(8 + 6h + 12h^2 + h^3) + 3 - 16 - 3}{h}$   
 $= \frac{16 + 12h + 24h^2 + 2h^3 - 16}{h}$   
 $= \frac{h(12 + 24h + 2h^2)}{h} = 12 + 24h + 2h^2$

Instantaneous:  $v(2) = s'(2) = \lim_{h \rightarrow 0} \frac{s(2+h) - s(2)}{(2+h) - 2} = \lim_{h \rightarrow 0} (12 + 24h + 2h^2) = 12 + 24 \cdot 0 + 2 \cdot 0^2 = 12$   $\square$

38.  $s(t) = \frac{16}{t^2} - \frac{4}{t}$

Average:  $\frac{s(2+h) - s(2)}{(2+h) - 2} = \frac{(\frac{16}{(2+h)^2} - \frac{4}{2+h}) - (\frac{16}{2^2} - \frac{4}{2})}{h} = \frac{(2+h)^2}{(2+h)^2} = \frac{16 - 4(2+h) - (4-2)(2+h)^2}{h(2+h)^2}$   
 $= \frac{16 - 8 - 4h - 2(4 + 4h + h^2)}{h(2+h)^2} = \frac{8 - 4h - 8 - 8h - 2h^2}{h(2+h)^2}$   
 $= \frac{-h(12 + 2h)}{h(2+h)^2} = \frac{-(12 + 2h)}{(2+h)^2}$

Instantaneous:  $v(2) = s'(2) = \lim_{h \rightarrow 0} \frac{s(2+h) - s(2)}{(2+h) - 2} = \lim_{h \rightarrow 0} -\frac{12 + 2h}{(2+h)^2} = -\frac{12 + 2 \cdot 0}{(2+0)^2} = -3$   $\square$