

RATES OF CHANGE

Sam: The position of a train is given by $s(t) = 3t^3 - 8t$ miles where t is measured in hours. Supposing the positive direction is east, determine the direction the train is moving when...

(A) we begin tracking the train's movement.

(B) when the train is at position 0.

(C) when the train has acceleration 0.

Sol: $S(t) = 3t^3 - 8t$, $v(t) = S'(t) = 9t^2 - 8$, $a(t) = v'(t) = 18t$

A: we begin tracking at time $t=0$.

$$v(0) = 9 \cdot 0^2 - 8 = -8 < 0.$$

\therefore the train is moving west.

B: Train is at position 0 when $S(t) = 0$

$$3t^3 - 8t = 0, \text{ i.e. } t(3t^2 - 8) = 0, \text{ i.e. } t = 0 \text{ or } 3t^2 - 8 = 0$$

$$\text{i.e., } t = 0 \text{ or } t^2 = \frac{8}{3} \text{ i.e., } t = 0 \text{ or } t = \pm \sqrt{\frac{8}{3}}$$

Reject negative \uparrow

$$v(0) = -8 < 0 \text{ and } v\left(\sqrt{\frac{8}{3}}\right) = 9\left(\sqrt{\frac{8}{3}}\right)^2 - 8 = 9 \cdot \frac{8}{3} - 8 = 16 > 0$$

\therefore at $t = \sqrt{\frac{8}{3}}$ the train is moving east.
at $t = 0$ the train is moving west.

C: $a(t) = 0$ when $18t = 0$ i.e. $t = 0$.

\therefore moves west when $a(t) = 0$ (by part A).



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Qam: The position of a train is given by $s(t) = t^3 - 8t$ miles where t is measured in hours. Supposing the positive direction is east, determine the direction the train is moving when...

- (A) we begin tracking the train's movement.
- (B) when the train is at its initial position.
- (C) when the train has acceleration 0.

Sol: $S(t) = t^3 - 8t$, $v(t) = S'(t) = 3t^2 - 8$, $a(t) = v'(t) = 6t$

A: we begin tracking at time $t=0$.

$$v(0) = 3 \cdot 0^2 - 8 = -8 < 0.$$

\therefore the train is moving west.

B: Train is at position 0 when $S(t) = 0$

$$t^3 - 8t = 0, \text{ i.e. } t(t^2 - 8) = 0, \text{ i.e. } t = 0 \text{ or } t^2 - 8 = 0$$

$$\text{i.e., } t = 0 \text{ or } t^2 = 8 \quad \text{i.e., } t = 0 \text{ or } t = \pm\sqrt{8}$$

Reject negative \uparrow

$$v(0) = -8 < 0 \quad \text{and} \quad v(\sqrt{8}) = 9(\sqrt{8})^2 - 8 = 9 \cdot 8 - 8 = 64 > 0$$

\therefore at $t = \sqrt{8}$ the train is moving east.
at $t = 0$ the train is moving west (by part A).

C: $a(t) = 0$ when $6t = 0$ i.e. $t = 0$.

\therefore moves west when $a(t) = 0$ (by part A).

